Analysis of Safety Parameters with usage of criticality analysis and heat output for Pressurized water reactors (Case Study: TRIGA MARK 2 Nuclear Reactor

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Abstract

Analysis of safety parameters with usage of criticality analysis and heat output for pressurized water reactors (Case Study: TRIGA MARK 2 Nuclear Reactor). The objective of the project is to couple critically equations for neutron population inside the nuclear core of PWR reactor with the heat outputs and to create a control program which analyses safety of the PWR reactor.

- FORTAN is used as the main programming language for the analysis of neutron population and fission critically equations..
- Analysis of Neutron Population, Cross sections and its effect on criticality is performed for sub-critical, super-critical and critical reactive states.
- ANSYS CFD is used for the analysis of heat transfer from nuclear fuel rods to water under high pressure conditions for temperature profile of various reactive states.

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CHAPTER 1

INTRODUCTION

There has been continuous changes and development in the field of energy in the 20^{th} century. The development range is across the whole nation including urban as well as the rural. The work rate has increased due to advancement in technology, systematic work culture, energy source utilization and using alternate sources of energy.

Today Petroleum Industries have become the backbone of the nation's economy and each and every industry is dependent on it. Studies indicate that the petroleum based products are exhausting day by day with an increase in the concern for what could replace it at that scale. Governments of various nations have started to increase their funds on alternate source of energy which could be as abundant as petroleum once was. Moreover petroleum has its own losses due to increased pollution, which has caused global increase in temperatures, global warming.

In India the rate of production of petroleum and its products is much less than its requirements. Thus we often depend on other nations for our fuel requirements such as the United States of America, Southern Arab Countries etc. But this overreliance on imported fuel may lead to increase in debt and restricted assets. So it's important to think on various other types of energy required to fulfill our demands.

The coal reserves in the world would maximum serve for 40 years and if crude oil could be replaced by it, its life would have to increase a few more years (around 20-30) which is less than possible. Thus the best option with us is to develop nuclear power sources to completely eliminate the use of crude or coal based thermal energy/

It is important to take into consideration the cost of generation of energy during the selection because if all the nuclear fuel if being imported would increase to cost of generation much higher and that could affect the economy of the nation. So various tests need to be done on other kinds of nuclear energy which can be harnessed, one of which is using Thorium.

In this project we study the safety parameters required for such nuclear reactors with the help of nuclear criticality analysis and heat output of a pressurized water reactor. We use the parameters of a specific TRIGA MARK II reactor and study the neutron flux generation for safe criticality limit. We also study the relationship between heat output and the neutron generation to find the safe limit of operation.

CHAPTER 2

REVIEW OF LITERATURE

2.1 General Description of Reactor

250-kW TRIGA Mark II reactor uses light water and graphite rods as reflector. Various views of the reactor are shown in Figures 2.1 and 2.2.

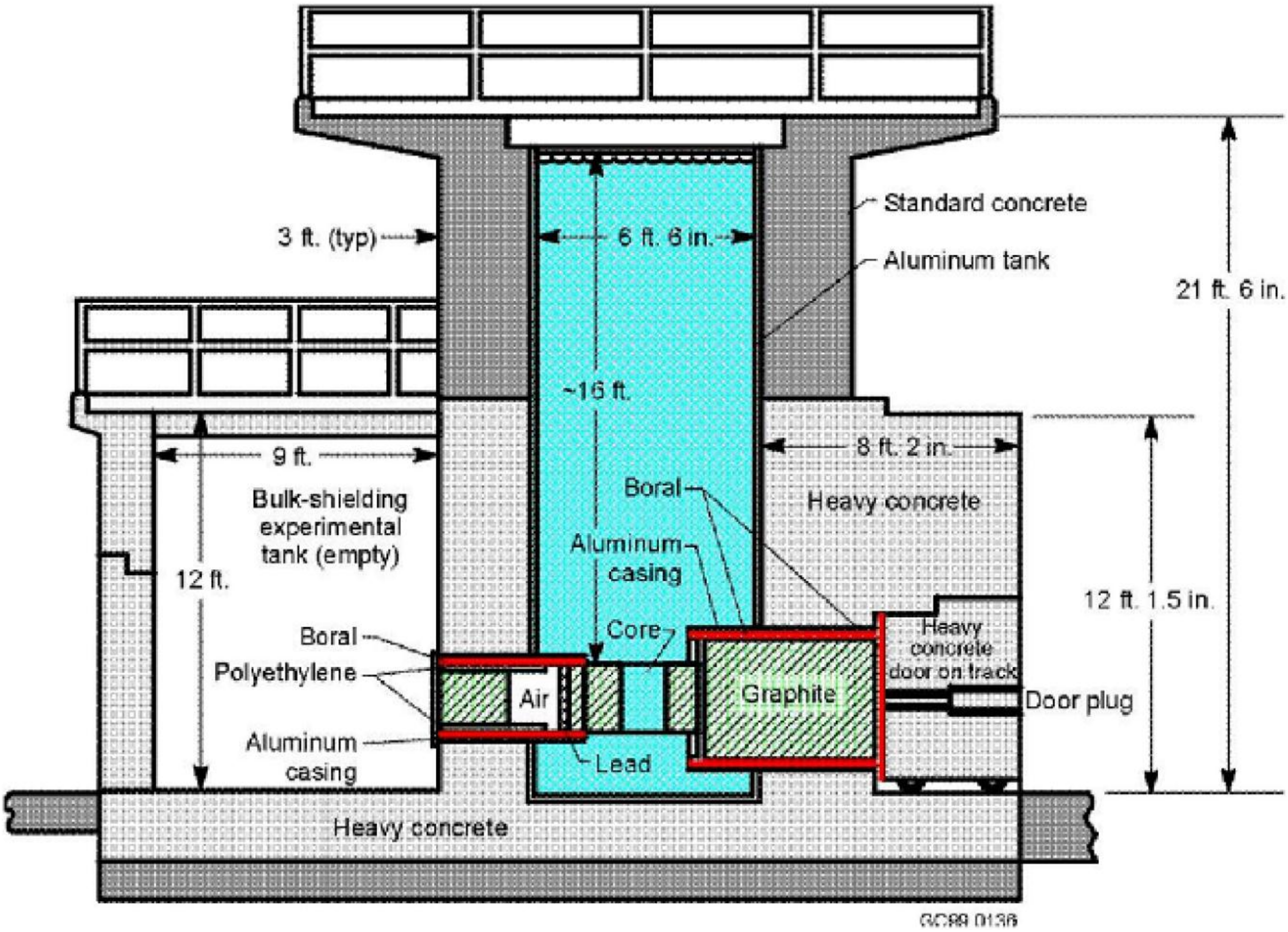

Figure 2.1: TRIGA Reactor Side View

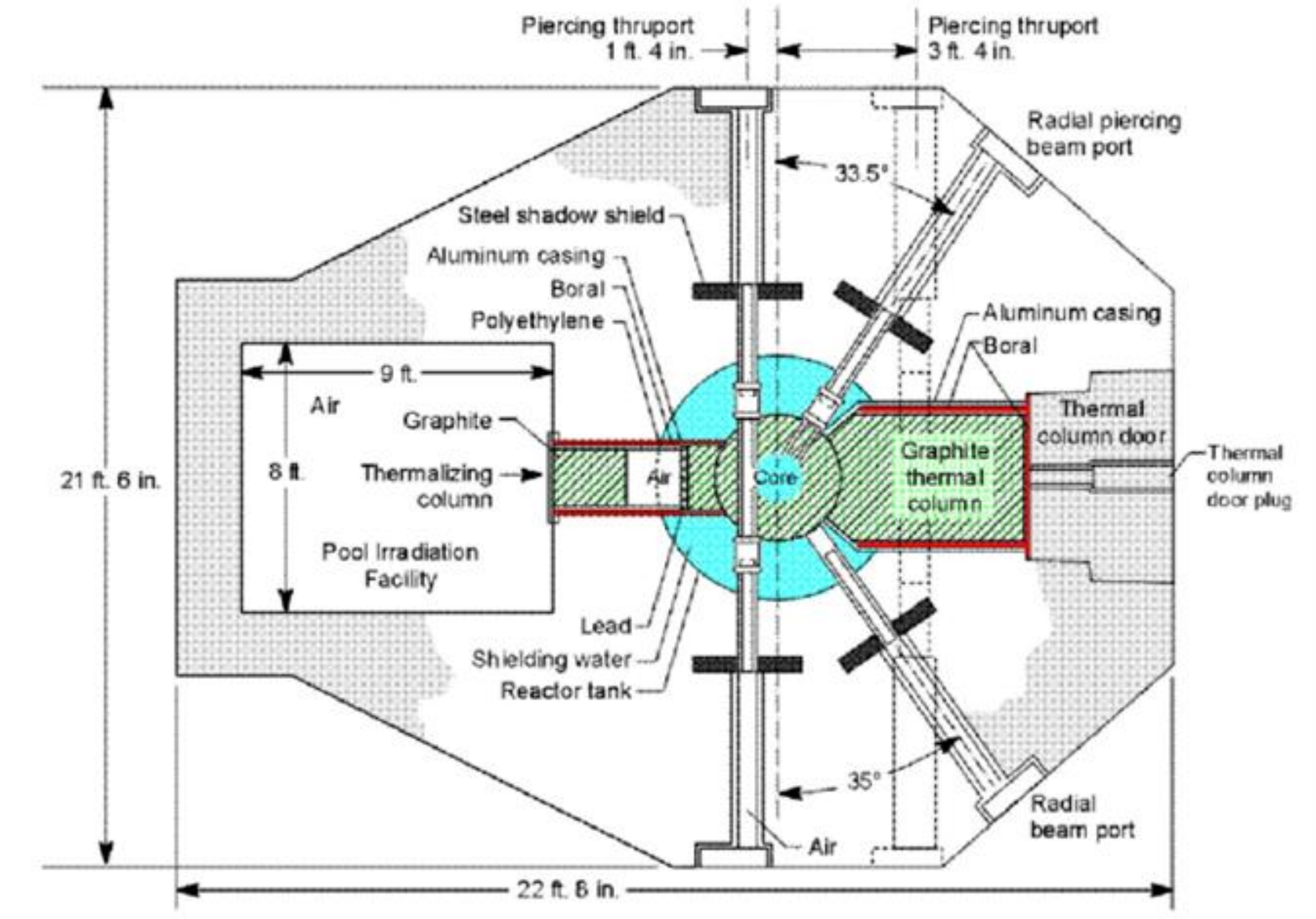

Figure 2.2 TRIGA Reactor Top View

2.2 Reactor Core

A cylindrical core structure is placed at the bottom. Fig. 2.3 shows the arrangement of the core.

The core consists of 91 rods which include fuel elements and other components like a neutron source, control rods, irradiation channels, etc.

All elements form a concentric circular sings with their actual position and diameters mentioned in Table 2.1.

Aluminium makes up the empty irradiation tube which takes the position of A1. E10, D8, E11 positions are taken by the fuel rods.

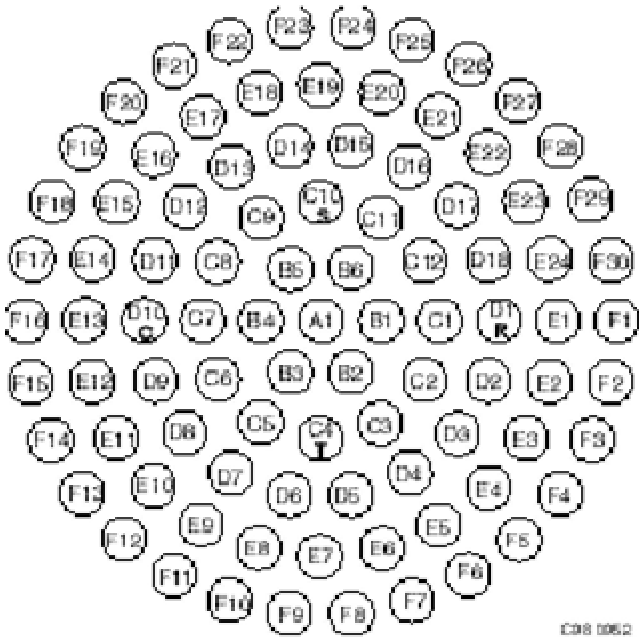

Figure 2.3: Core rod arrangement

Table 2.1 Ring Element configuration

Ring name	Position	Dia (inches)
A	1	0
B	6	3.2
C	12	6.3
D	18	9.4
E	24	12.5
F	30	15.7

2.3 Core Support Grids

They are aluminium rods of same thickness and varying diameters that act as a support structure of the fuel rods. To provide where the actual rod will go in, there is a top grid plate provided for correct positioning. Flux in the core is measure using probes which are instered from the small holes on the upper plate. They are in radial direction with around three radial lines containing 16 holes from B-3 to F-11, B-6 to F-24 and B-2 to F-9. Fig. 2.4 explains the above arrangement.

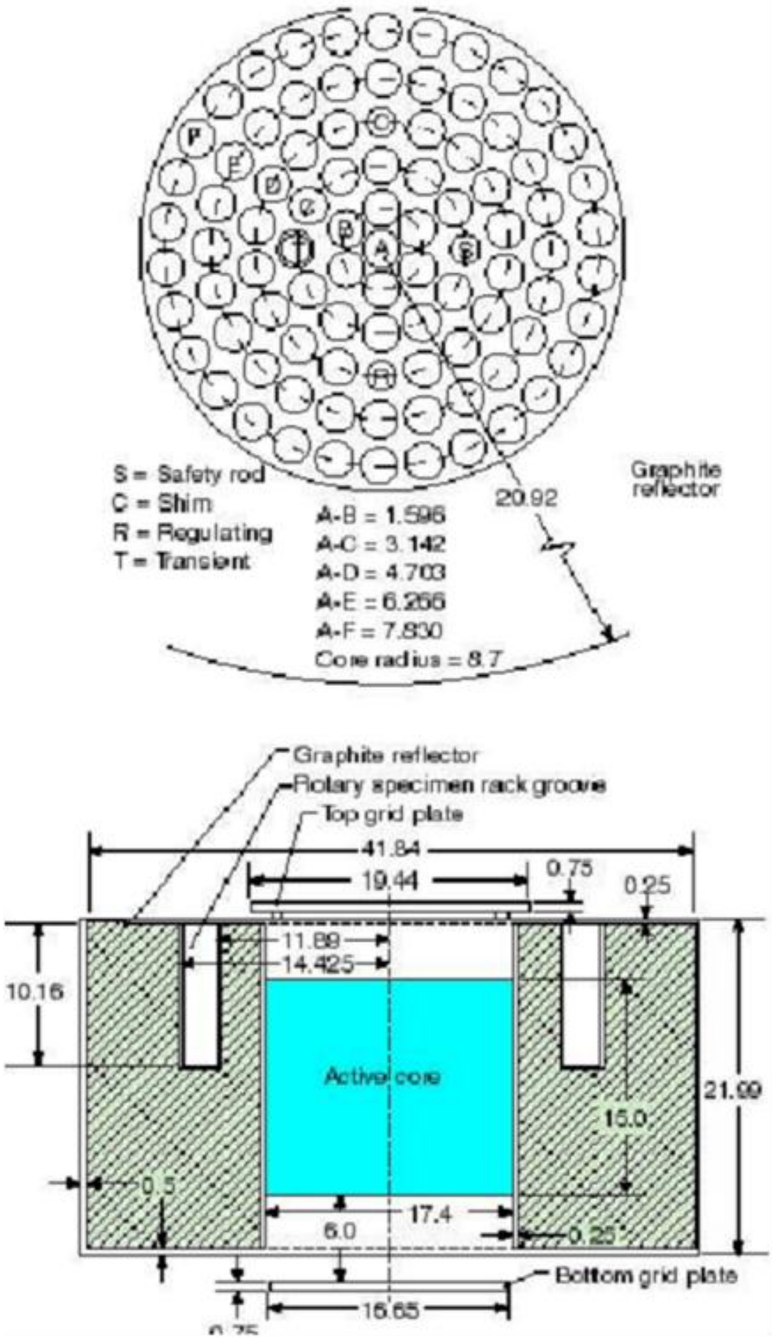

Figure 2.4: Schematic Views

2.4 Graphite Reflector

The dimensions of the casing, the rotary specimen-rack and the reflector groove are provided in Figure 2.4 and in Table 2.2.

Table 2.2: Reflector Dimensions

Component	**Dimension [inch]**	**Material**
Reflector		Graphite/Carbon
Diameter – Outer Diameter – Inner	42.8 18.9	
Cladding		Al
Inner – Top Thickness Outer – bottom Thickness	0.15 0.30	
Rotary specimen-rack		Air
Diameter – Outer Diameter – Inner	24.9 25.8	
Specimen rack		Aluminum
Diameter of holes	32 mm	
Irradiation channels		Aluminum
Diameter – Outer Diameter – Inner	8.1 6.4	

2.5 Fuel Elements

Cylindrical fuel elements have a stainless steel cladding of specification SS-304. It has a diameter of 1.5 inch with length of 28 inches. Inside these casings the fuel element length is 15 inches with graphite slugs at both the opening ends. These act as reflectors. A homogeneous mixture of zirconium hydride and uranium form the fuel. The geometry and dimensions are given in Fig. 2.5 and Table 2.3.

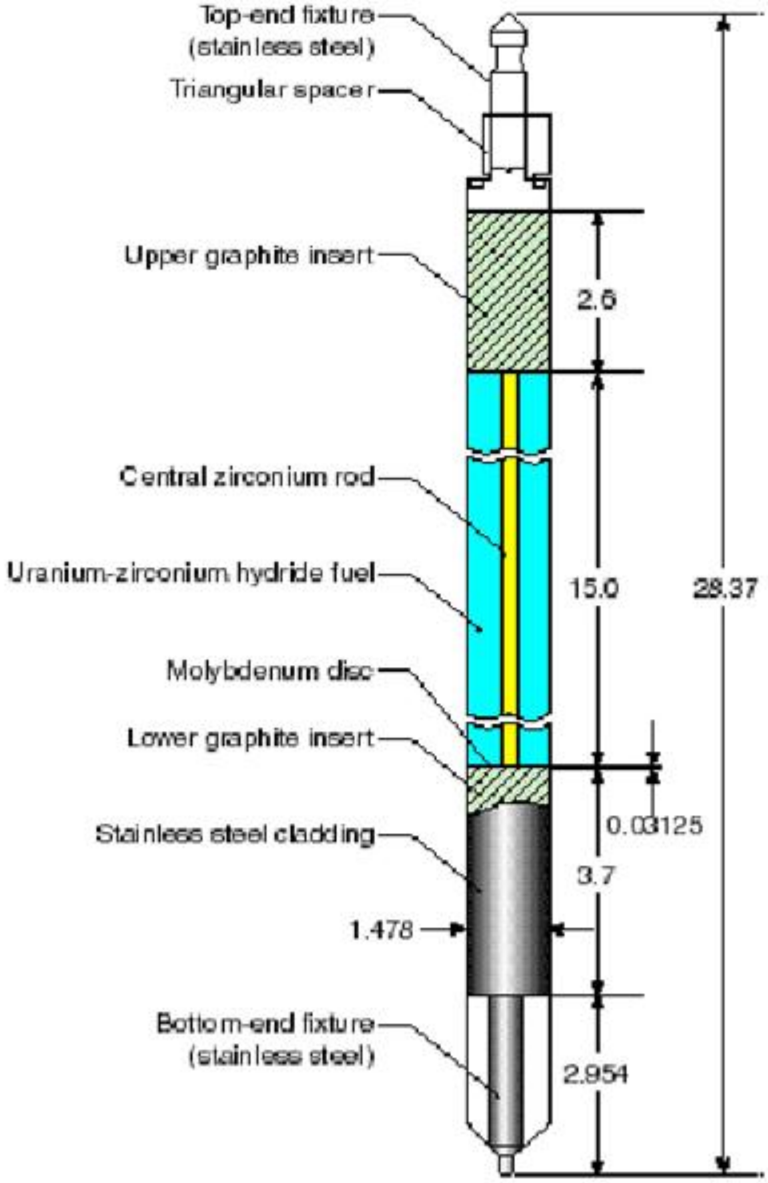

Figure 2.5: Fuel Element

Table 2.3: Data for Fuel-Element

Component	Dimension [inch]	Material	Density [g/cm3]
Fuel element			
Diameter - Outer Element - length Material of the Fuel	1.5 28.5	U-ZrH	6.1
Diameter - Outer Diameter - Inner	1.5 0.24		
Zr rod		Zr	6.4
Diameter Height	0.24 15.0		
Axial reflector		Graphite	1.4
Diameter Height upper Height lower	1.5 2.7 3.8		
Supporting disc		Molybdenum	10.3
Thickness	0.03225		
Cladding		SS-304	7.8
Thickness	0.03		
Top and bottom ends		SS-304	7.8
Height top Height bottom	4.2 3.1		

CHAPTER 3

METHODOLOGY

3.1 Diffusion Theory

It is necessary to predict neutron distr ibution inside a nuclear reactor. The exact description of all the processes of neutrons (collisions, transport, nuclear reactions) is very difficult. The first approximation describes the movement of neutrons as a kind of diffusion. This approximation is called diffusion approximation and was used in development of the first types of nuclear reactors. More advanced methods are developed now, but still, diffusion theory is widely used for the analysis of large nuclear reactors. The complete theory describing all neutron properties with little approximation is Transport theory solving Boltzmann transport equation.

3.1.1 Fick's Law

The solute diffuses from the region of higher concentration tothe region of lower concentration when concentration of a solute in one region of solution is greater than in another. The negative gradient of the solute concentration gives rate of diffusion. Neutrons behave to a good approximation in the same way.This concept remains the same for neutrons in a reactor where the neutron population of one region is more than other.

3.1.2 Neutron Current Density

If neutron density varies along x-direction, the net flow of neutrons perpendicular to the x-direction through an unit area per unit area can be expressed as:

$$J_x = -D\frac{d\phi}{dt} \tag{3.1}$$

The flux is generally function of three spatial variables, therefore:

$$J = -D grad \phi = -D \bigtriangledown \phi \tag{3.2}$$

Here J is called neutron current density

3.1.3 Diffusion Coefficient

We assume that D is not a function of spatial variables. The diffusion coefficient can be approximately calculated as:

$$D = \frac{\lambda_{tr}}{3} \tag{3.3}$$

where λ_{tr} is transport mean free path.

$$\lambda_{tr} = \frac{1}{\Sigma_{tr}} = \frac{1}{\Sigma_s(1 - \overline{\mu_s})} \tag{3.4}$$

Mean free path is an average distance a neutron will move in its original direction after infinite number of collisions. It can be calculated for most of the neutron energies as:

$$\overline{\mu} = \frac{2}{3A} \tag{3.5}$$

3.1.4 Validity of Ficks Law

Fick's law is approximation which is not valid under the following conditions:

- Strongly absorbent Neutron Medium

• Strongly anisotropic scattering of neutrons.

These conditions are acceptable for every practical reactor problem. Ficks law and diffusiontheory is therefore only the first estimate.

3.1.5 General Equation of Continuity

In an arbitray volume of a diffusive medium the number ofneutrons may change. The change of the number of neutrons is a result of a flow of neutrons in or out of V, some neutrons are absorbed inside V and there might be also neutron sources inside volume V.

$$\begin{pmatrix} \text{rate of change} \\ \text{in number of} \\ \text{neutrons in V} \end{pmatrix} = \begin{pmatrix} \text{rate of} \\ \text{production} \\ \text{of neutrons} \\ \text{in V} \end{pmatrix} - \begin{pmatrix} \text{rate of} \\ \text{absorption} \\ \text{of neutrons} \\ \text{in V} \end{pmatrix} - \begin{pmatrix} \text{rate of} \\ \text{leakage of} \\ \text{neutrons} \\ \text{from V} \end{pmatrix}$$

3.1.6 Rate of Change of Neutrons in V

Total number of neutron can be given by:

$$\int_V n dV$$

The rate of change is:

$$\frac{d}{dt} \int_V n dV$$

which is written as:

$$\int_V \frac{\partial n}{\partial t} dV$$

3.1.7 Production and Absorption Rate in V

s - rate at which neutrons source emits per volume in V. Thus,

$$\text{Production ratre} = \int_V s dV$$

$\Sigma_a \phi$ is the rate at which neutrons are lost by absorption per cm^3/sec.

$$\text{Absorption rate} = \int_V \Sigma_a \phi dV$$

3.1.8 Leakage Rate out of V

The current density is given by J. Thus the total leakage can be given by:

$$\text{Leakage Rate} = \int_A \mathbf{Jn} dA$$

By the divergence theorem:

$$\int_A \mathbf{Jn} dA = \int_V div \mathbf{J} dV$$

3.1.9 Resulting Equation of Continuity

$$\int_V \frac{\partial n}{\partial t} dV = \int_V s dV - \int_V \Sigma_a \phi dV - \int_V div \mathbf{J} dV$$

$$\frac{\partial n}{\partial t} = s - \Sigma_a \phi - div\mathbf{J} \tag{3.6}$$

$$-div\mathbf{J} - \Sigma_a \phi + s = 0$$

3.1.10 Diffusion Equation

There is a relation between neutron flux and neutron current density. One of these unknowns can be eliminated by Ficks law. Substitution of (2.2) into (2.6) leads to:

$$-div(-Dgrad\phi) - -\Sigma_a \phi + s = \frac{\partial n}{\partial t}$$

Diffusion coefficient (D) is spatially independent

$$Ddiv(grad\phi) \quad \Sigma_a \phi + s = \frac{\partial n}{\partial t}$$
$$-$$

The continuity equation can be fur ther simplified by introducing symbol

$$\bigtriangledown^2 \equiv \text{div grad}$$

called Laplacian operator.

The resulting equation is called diffusion equation

$$D.\bigtriangledown^{2\cdot}\phi - \Sigma_{a.}\phi + s = \frac{1}{v}\frac{\partial \phi}{\partial t} \tag{3.7}$$

If only time independent problems are considered, steady-state diffusion equation is formulated

$$D \nabla^2 \phi - \Sigma_a \phi + s = 0 \tag{3.8}$$

3.1.11 Laplacian Operator

Formula for Laplacian depends on used coordinate system.

- in rectangular coordinates:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

- for cylindrical coordinates:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial \vartheta^2} + \frac{\partial^2}{\partial z^2}$$

- in spherical coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2 \sin\vartheta}\frac{\partial}{\partial \vartheta}(\sin\vartheta \frac{\partial}{\partial \vartheta}) + \frac{1}{r^2 \sin^2\vartheta}\frac{\partial^2}{\partial \varphi^2}$$

3.1.12 1D Laplacian Operators

In the simplest examples in one-dimensional space, the Laplacian operator reduces to the following formulas:

- rectangular coordinates: $$\nabla^2 = \frac{\partial^2}{\partial x^2}$$

- cylindrical coordinates: $$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

- for spherical coordinates: $$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

3.1.13 Diffusion Length

The equation 2.8 is often divded by D, resulting in:

$$\bigtriangledown^2 \phi - \frac{1}{L^2}\phi + \frac{s}{D} = 0 \tag{3.9}$$

Parameter L^2 is defined as:

$$L^2 = \frac{D}{\Sigma_a}$$

The quantity L is called diffusion length with unit cm. Quantity L^2 is diffusion area, unit cm^2.

3.1.14 Boundary Condition

The diffusion equation was derived using Ficks law, therefore conditions for validity of Ficks law are also valid for the diffusion equation. There are typical boundary conditions:

- Neutron flux must be non-negative and finite:

$$0 \leq \phi < \infty$$

$${}_A = {}_B$$

$$(J_A)_n = (J_B)_n$$

- Boundary condition for an external boundary of a diffusive medium.
- Source conditions.

3.1.15 Extrapolation Distance

Ficks law is not valid for area close to an external surface between the diffusive medium and atmosphere. It was found that if the flux vanishes in a distance d from the surface, then the flux calculated by diffusion theory is close to the real flux. From relation for diffusion coefficient D = λ_{tr}/3 results that d = 2.13 D. The extrapolation distance is usually in units of several cm and therefore it can be in many cases neglected and assumed that neutron flux diminishes at the physical boundary.

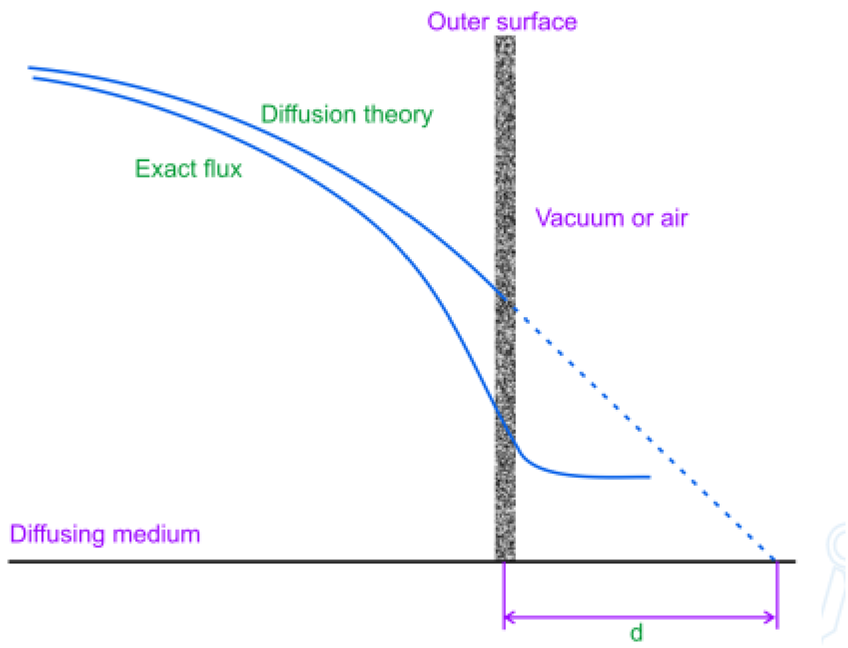

Figure 3.1: Extrapolation Distance Visualisation

If d is not negligible, physical dimensions of the reactor are increased by d and extrapolated boundary is formulated with dimension $a + d$.

Source Condition The diffusion equation is not valid for the neutron source location, but it is necessary to match the magnitude of the neutron flux to the source intensity. The source is surrounded by area for which it holds that all neutrons flowing through this area must come from the neutron source characterized by source emissivity S. Formulation depends on the source geometry: planar (3-10a), point (3-10b), or line (3-10c)

$$\lim_{x \to 0} J(x) = \frac{S}{2} \tag{3.10a}$$

$$\lim_{r \to 0} 4\pi r^2 J(r) = S \tag{3.10b}$$

$$\lim_{r \to 0} 2\pi r J(r) = S \tag{3.10c}$$

This condition will be illustrated by examples of neutron sources in diffusive media.

3.1.16 Infinite Diffusive Medium

Spatial distr ibution of neutron flux in an infinite media will be calculated using diffusion equation and boundar y conditions. Basic source geometries will be calculating plane, point and line. Only monoenergetic sources of neutrons are analysed

3.1.17 Planar Source in Infinite Diffusive Medium

The flux is only function of distancefrom the plane, e.g. in x direction. The source location is not part of the analysed area. Theplane is located in x=0 and there are two solution for positive (x>0) and negative (x<0)directions. The diffusion equation (5-9) has the following form:

$$\frac{d^2\phi}{dx^2} - \frac{1}{L^2}\phi = 0$$
$$x \neq 0$$

Solution for x=0 is expected in form:

$$\phi(x) = e^{-\lambda x}$$

Second Derivative:

$$\frac{d^2\phi}{dx^2} = \lambda^2 e^{-\lambda x}$$

Substituted in the above equation leads to:

$$\lambda^2 e^{-\lambda x} = \frac{1}{L^2} e^{-\lambda x}$$

There are two possible solution for $\lambda = \pm\frac{1}{L}$ The diffusion equation for planar source has thus general solution with two constants to be determined by boundary conditions

$$\phi(x) = Ae^{-x/L} + Ce^{x/L}$$

The constant A is determined from the source condition. From Ficks law:

$$J = -D\frac{d\phi}{dx} = \frac{DA}{L}e^{-x/L}$$

Source condition for the planar source:

$$\lim_{x\to 0} J(.x) = \lim_{x\to 0}\frac{D.A}{L}e^{-x./L} = \frac{S}{2}$$

This gives constant A:

$$A = \frac{SL}{2D}$$

Therefore,

$$\phi(x) = \frac{SL}{2D}e^{-x/L} \tag{3.11}$$

The solution is valid for x>0, but because of symmetry of the problem similar formulation could be obtained for negative x-direction.

3.1.18 Point Source in Infinite Diffusive Medium

The sourceis located in the centre of a spherical coordinate system and neutron flux depends only ondistance r from the source. The diffusion equation (3.9) in spherical coordinate systembecomes for $r \neq 0$:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{1}{L^2}\phi = 0$$

Substituting $u(r) = r\phi(r)$

Substitution into the above equation results in:

$$\frac{d^2u}{dr^2} - \frac{1}{L^2}u = 0$$

The solution for function u is found in an identical way as in the case of the planar source:

$$u(r) = Ae^{-r/L} + Ce^{r/L}$$

Transform to the original function gives:

$$\phi(r) = A\frac{e^{-r/L}}{r} + C\frac{e^{r/L}}{r}$$

Constants A and C must be determined from boundary conditions. It is clear that if the neutron density flux must remain finite, C must equal zero.

From Ficks law:

$$J = -D\frac{d\phi}{dr} = DA(\frac{1}{rL} + \frac{1}{r^2})e^{\ \ r/L}$$

The source condition for point source is:

$$\lim_{r \to 0} 4\pi r^2 J(r) = \lim_{r \to 0} 4\pi DA(\frac{r.}{L} + 1)e^{-r./L} = S.$$

This gives constant A:

$$A = \frac{S}{4\pi D}$$

Resulting equation for neutron flux distribution is following:

$$\phi(r) = \frac{Se^{-r/L}}{4\pi Dr} \tag{3.12}$$

3.1.19 Line Source in Infinite Diffusive Medium

The sourceis located in the centre of a cylindical coordinate system and neutron flux depends only ondistance r from the source. The diffusion equation (2.9) in the cylindical coordinate systembecomes for $r \neq 0$:

$$\frac{d^2\phi(r)}{dr^2} + \frac{1}{r}\frac{d\phi(r)}{dr} - \frac{1}{L^2}\phi(r) = 0$$

The equation can be transformed by substitution u=r/L into modified Bessels equation of the order zero:

$$u^2\frac{d^2\phi(r)}{du^2} + u\frac{d\phi(r)}{du} - u^2\phi(u) = 0$$

3.1.19.1 Odinary Bessel's Functions

Bessels equation is:

$$x^2\frac{d^2\phi}{dx^2} + x\frac{d\phi}{dx} + (\alpha^2x^2 - n^2)\phi = 0$$

where α and n are constants. 'n' is order of the equation, in practical problems it is usually zero General solution to Bessels equation is:

$$\phi(x) = AJ_n(\alpha x) + CY_n(\alpha x)$$

Functions Jn and Yn are called ordinary Bessels functions of the first and second kind, respectively.

3.1.19.2 Modified Bessel's Function

If α^2 is negative, Bessels equation becomes:

$$x^2\frac{d^2\phi}{dx^2} + x\frac{d\phi}{dx} - (\alpha^2x^2 + n^2)\phi = 0$$

General solution is in form of modified Bessels functions of the first and second kind, respectively $-I_n$ and K_n.

$$\phi(x) = AI_n(\alpha x) + CK_n(\alpha x)$$

This shows the Bessel's Function used in reactor physics.

General solution of this kind of Bessels equation is in form of modified Bessels function of the first (I) and second (K) kind of the order zero.

$$\phi(u) = AI_0(u) + CK_0(u)$$

$$\phi(r) = AI_0(r/L) + CK_0(r/L)$$

Given the fact that function

$$I_0 \to \infty$$

for

$$r \to \infty$$

, constant A must equal 0 and above equation reduces to:

$$\phi(r) = CK_0(r/L)$$

The constant C is found from the source condition.

From Ficks law, using $\frac{dK_0}{dr} = -K_1$:

$$J = -D\frac{d\phi}{dr} = -DC\frac{dK_0(r/L)}{dr} = \frac{DCK_1(r/L)}{L}$$

$K_1(x)$ behaves similar to $\frac{1}{x}$ function. It can be used in the source condition:

$$\lim_{r\to 0} 2\pi(r/L)DCK_1(r/L) = S$$

It can be written that $\lim_{r\to 0}[(r/L)K_1(r/L) = 1]$

Resulting value for constant C is:

$$C = \frac{S}{2\pi D}$$

Finally,

$$\phi(.r) = \frac{S}{2\pi D} K_0(\frac{r}{L}) \tag{3.13}$$

3.1.19.3 Cubical Reactor Geometry

We take a rectangular geometry of a,b and c, such that the flux vanishes at edges: $(\pm a/2, \pm b/2, \pm c/2)$

Rearranging equation 2.8,

$$D \bigtriangledown^2 \phi - \Sigma_c \phi + s = 0$$

Assume neutron source is: $s = \Sigma_c k \phi$ Assume $\phi(x, y, z) = F(x).G(y).H(z)$

Equation thus becomes:

$$G(y).H(z).\frac{\partial^2 F}{\partial x^2} + F(x)H(z)\frac{\partial^2 G}{\partial y^2} + F(x)G(y)\frac{\partial^2 H}{\partial z^2} = (\Sigma_c/D)(1-k)F(x).G(y).H(z)$$

Dividing out by $F(x)G(y)H(z)$ from both the sides yields:

$$(\frac{\partial^2 F}{\partial x^2})/F(x.) + (\frac{\partial^2 G}{\partial y^2})/G(y.) + (\frac{\partial^2 H}{\partial z^2})/H(z.) = (\Sigma_c/D)(1-k)$$

Given boundary conditions: F(a/2) = 0, F(x)= cos(xπ/a). Similarly: G(y) = cos(yπ/b), H(z) = cos(zπ/c)

$$(\frac{\partial^2 F}{\partial x^2})/F(x) = -(\pi/a)^2$$
$$(\frac{\partial^2 G}{\partial y^2})/G(y) = -(\pi/b)^2$$
$$(\frac{\partial^2 H}{\partial z^2})/H(z) = -(\pi/c)^2$$

Overall equation satisfies:

$$[(\pi./a)^2 + (\pi./b)^2 + (\pi./c)^2] = (\Sigma_c/D)(k-1)$$

Define Geometrical Buckling:

$$[(\pi./a)^2 + (\pi./b)^2 + (\pi./c)^2]$$

3.1.19.4 Cylindrical Reactor Geometry

Separation of variables in cylindrical geometry proceeds in similar fashion. Cylindrical dimensions: radius= R, Height= H Assume flux vanishes at: (R,±H/2)

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} - \frac{\Sigma_c}{D}\phi = \frac{-S}{D}$$

Assume: $\phi(r,z) = F(r)G(z)$

Equation becomes:

$$[\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r}]G(z) + \frac{\partial^2 G}{r^2}F(r) = \frac{-\Sigma_c}{D}(1-k)F(r)G(z)$$

Dividing out $F(r)G(z)$ from both sides yiels:

$$\frac{[\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r}]}{F(r)} + \frac{\frac{\partial^2 G}{r^2}}{G(z)} = \frac{-\Sigma_c}{D}(1-k)$$

Axial portion is similar to that of cubical geometry:

$$\frac{\frac{\partial^2 G}{r^2}}{G(z)} = \frac{-\pi}{H}^2$$

$$G(z) = \cos(\frac{z\pi}{H})$$

Radial portion involves Bessel Function of the first kind:

$$\frac{[\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r}]}{F(r)} = (\frac{-2.405r}{R})^2$$

8/12/15 11:44 AM MATLAB Command Window

```
>> X = 0:0.1:10;
Y = zeros(0,101);
i = 0
Y(i+1,:) = bessely(i,X);

i =

     0

>> plot(X,Y,'LineWidth',1.5)
axis([-0.1 10.2 -2 1])
grid on
legend('Y_0','Location','Best')
title('Bessel Functions of the Second Kind for v = 0')
xlabel('X')
ylabel('Y_v(X)')
>> hold on
>> J = zeros(0,101);
i = 0
J(i+1,:) = besselj(i,X);

i =

     0

>> plot(X,J,'LineWidth',1.5)
axis([-0.1 10.2 -2 1])
grid on
legend('J_0','Location','Best')
title('Bessel Functions of the First Kind for v = 0')
xlabel('X')
ylabel('J_v(X)')
>>
```

Figure 3.2: Matlab Code to Solve Bessel Function

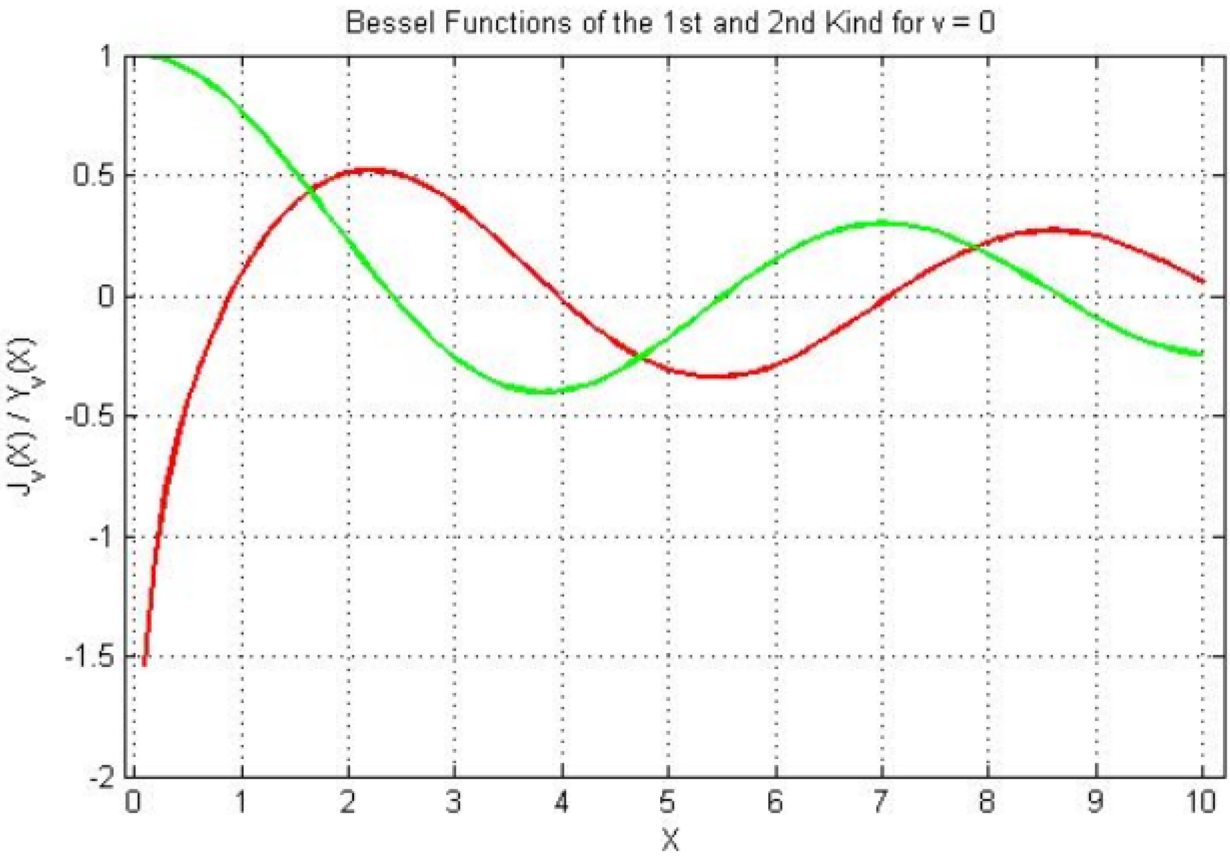

Figure 3.3: Odinary Bessel Function Solution

```
MATLAB Command Window

>>  X = 0:0.01:5;
Y = zeros(0,51);
i = 0;
I(i+1,:) = besseli(i,X);
plot(X,I,'LineWidth',1.5)
hold on
axis([0 5 0 5])
grid on
syms x y
>> for nu = [0]
ezplot(besselk(nu, x))
hold on
end
```

Figure 3.4: Matlab Code to Solve Modified Bessel Function

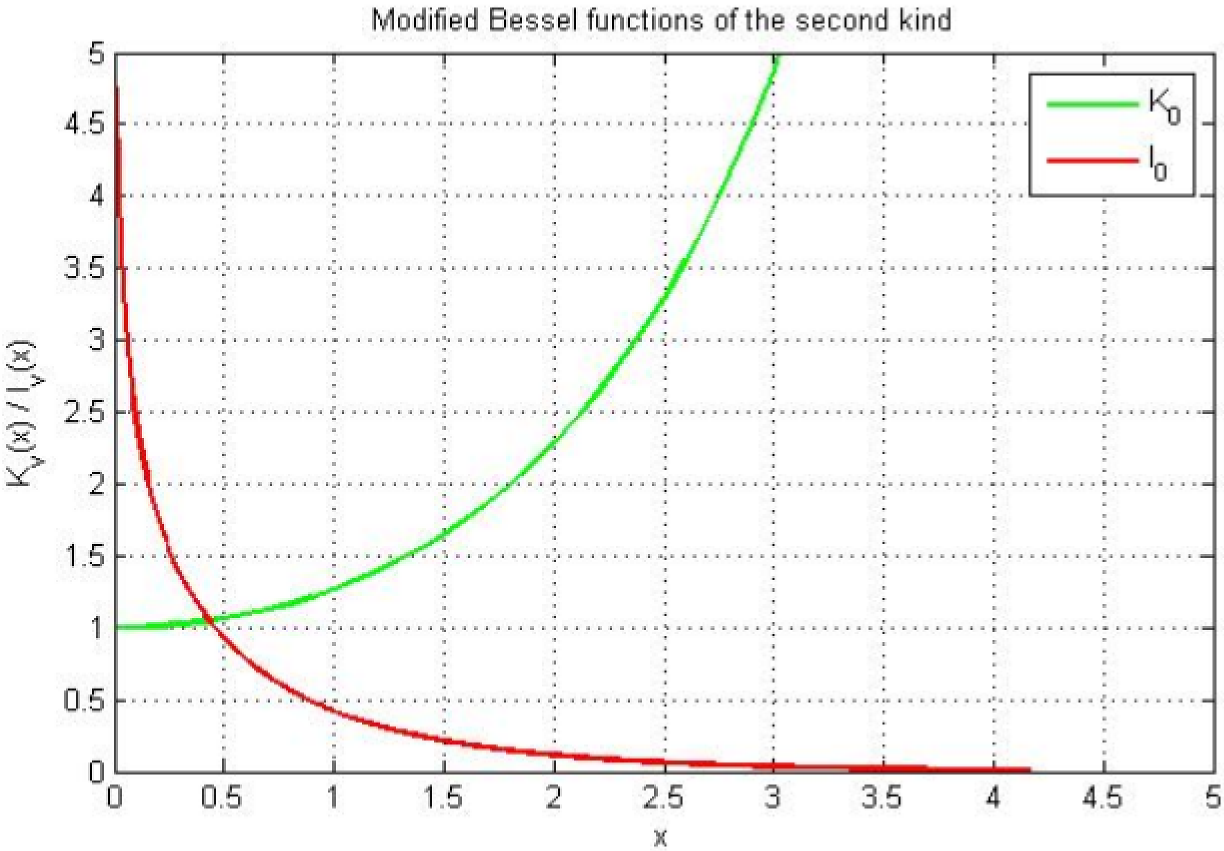

Figure 3.5: Modified Bessel Function Solution

MATLAB Command Window

```
>>  x = -5:0.01:5;
a = 10;
X = cos(x.*pi./a);
plot(x,X);
>> y = -2:0.01:2;
>> b = 4;
>> Y = cos(y.*pi./b);
>> plot(y,Y);
>> z = -3:0.1:3;
>> c = 6;
>> Z = cos(z.*pi/c);
>> grid on;
>> grid on;
>> grid on;
>>
```

Figure 3.6: Matlab function for Cubical Geometry

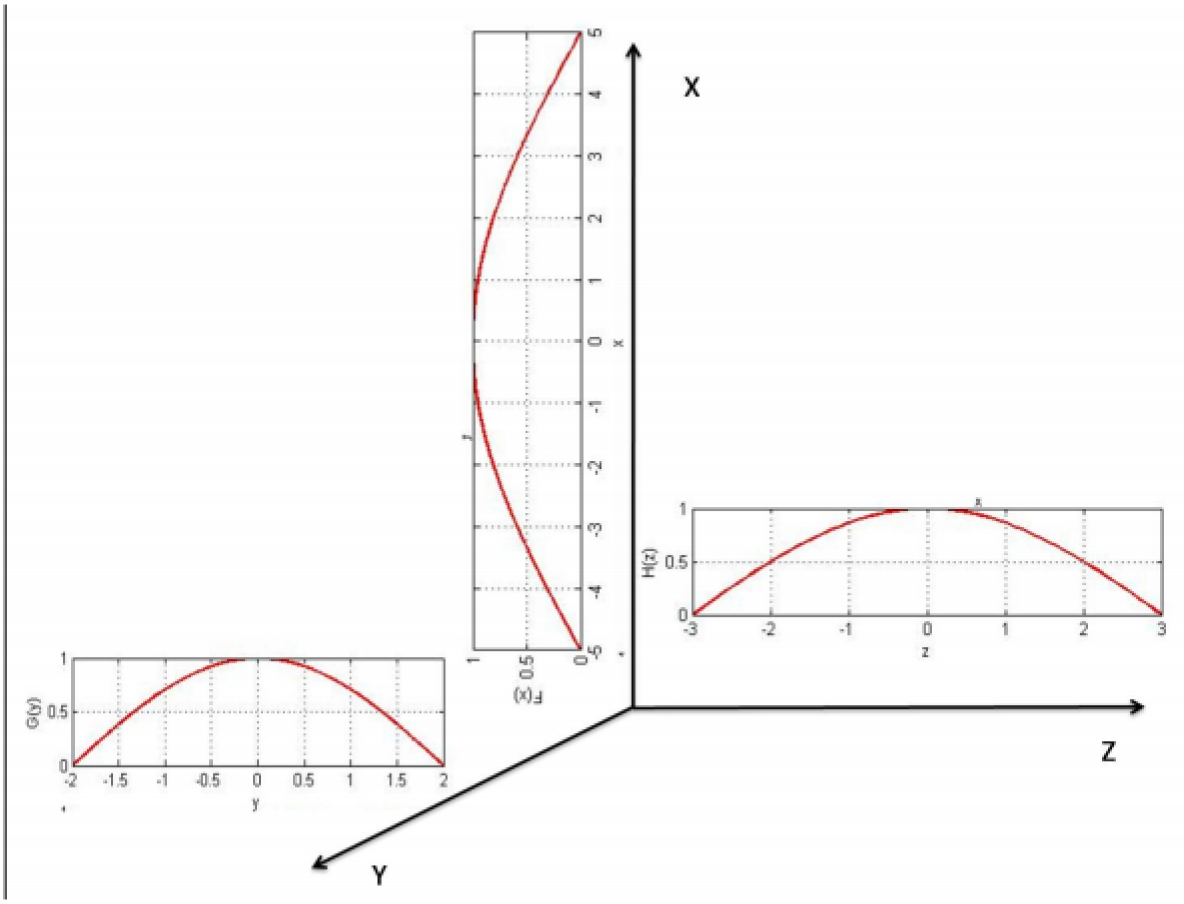

Figure 3.7: Neutron Flux in a Cubical Geometry

$$F(r) = J_0 \frac{2.405r}{R}$$

Overall Geometric Buckling for a cylinder is expressed as:

$$B^2 = (\frac{2.405}{R})^2 + (\frac{\pi}{H})^2$$

3.2 Neutron Multiplication Factor

3.2.1 Fission Source of Neutrons

The reactor is described by one-group diffusion equation:

$$D \bigtriangledown^2 \phi - \Sigma_a \phi + s = \frac{1}{v} \frac{\phi}{\partial t}$$

This equation is time-dependent and power of the reactor might increase or decrease.

Fission neutrons are the source of neutrons (s) in a nuclear reactor.

If Σ_f is fission cross-section of the fuel and number of neutrons emitted per one fission, source s can be expressed as:

$$s = v \Sigma_f \phi$$

If fission source does not balance neutron absorption and leakage, then right-hand side of equation is nonzero.

Parameter k can be used to adjust the source strength and to reach a steady state diffusion equation:

$$D \bigtriangledown^2 \phi - \Sigma_a \phi + \frac{1}{k} v \Sigma_f \phi = 0$$

3.2.2 One-Group Reactor Equation

Quantity geometric buckling (B^2) is defined as:

$$B^2 = \frac{1}{D}(\frac{v}{k}\Sigma_f - \Sigma_a)$$

Then previous equation can be rewritten in form of one-group reactor equation: $\bigtriangledown^2\phi + B^2\phi = 0$

The formula for buckling can be solved for the constant k:

$$k = \frac{v\Sigma_f}{DB^2 + \Sigma_a}$$

3.2.3 Multiplication Factor

$$k = \frac{v\Sigma_f}{DB^2 + \Sigma_a} = \frac{v\Sigma_f\phi}{DB^2\phi + \Sigma_a\phi} = \frac{v\Sigma_f\phi}{D\bigtriangledown^2\phi + \Sigma_a\phi}$$

Physical interpretation of the previous equation is following:

- Numerator is the number of neutrons born in fission in the currentgeneration.
- Denominator represents neutrons lost from the previous generation.
- Since all neutrons must be absorbed or leak from the reactor, thedenominator must be also born in the previous generation.

This is definition of multiplication factor for a finite reactor. It can be also defined as a neutron birth rate divided by sum ofneutron absorption and leakage rate.

3.2.4 Multiplication Factor for Infinite Reactor

The neutron source term can be rewritten with neutron absorption. Let aF be cross-section for neutron absorption in fuel, then: $s = \eta\Sigma_{aF}\phi$

Quantity η is called neutron reproduction factor and means number of produced neutrons per a single neutron absorbed in the fuel.

It can be further adjusted to:

$$s = \eta \frac{\Sigma_{aF}}{\Sigma_a} \Sigma_a \phi = \eta f \Sigma_a \phi$$

where f is $\frac{\Sigma_{aF}}{\Sigma_a}$

Quantity f is a fraction of neutrons absorbed from all neutrons absorbed in the reactor. In one generation, a certain number of neutrons is born related to , all these neutrons must be absorbedexpressed as $\Sigma_a \phi$.

Of these neutrons $f\Sigma_a\phi$ are absorbed in fuel and this leads to production of $\eta f \Sigma_a \phi$ neutrons.

We usually also consider the effect of fast electrons (ϵ) and the moderator to fuel ratio (p), thus the overall production of neutrons is of $\eta f p \epsilon \Sigma_a \phi$. All these neutrons must be again absorbed.

3.2.5 Multiplication Factor for Infinite Reactor

It means that absorption of $\Sigma_a\phi$ neutrons in one generation leads to absorption of $\eta f p \epsilon \Sigma_a \phi$ neutrons in the following generation. Absorption of neutrons is directly related to production of neutrons, therefore multiplication factor in an infinite reactor is defined as:

$$k_\infty = \frac{\eta f \epsilon p \Sigma_a \phi}{\Sigma_a \phi} = \eta f \epsilon p$$

η: multiplication constant (depends on the fuel and for uranium it is 1.32)
ϵ: fast fission factor (probability of fast neutrons undergoing fission) (For uranium it is 1.2, it increases with the diameter of the fuel rods)
p : moderator to fuel ratio (usually 0.874)
p : moderator to fuel ratio (usually 0.874)

3.2.6 Fortran Program for Calculation of Multiplication factor

NMF.f95*

```
PROGRAM Neutron
! Neutron Multiplication Factor
IMPLICIT NONE
REAl eff! multiplication constant (depends on the fuel)
REAl e  ! fast fission factor (probability of fast neutrons undergoing fission)
REAl p  ! moderator to fuel ratio (usually 0.874)
REAL N1 ! Number of atoms of fuel / cm^3
REAL N2 ! Number of atoms/cm^3 of moderator
REAL N3 ! Number of atoms/cm^3 of coolant
REAL S1 ! microscopic cross section for the fuel
REAL S2 ! microscopic cross section for the moderator
REAL S3 ! microscopic cross section for the coolant
REAL f  ! Thermal Utilization Factor
REAL k  ! Neutron Multiplication Factor
WRITE( *, '(A)', ADVANCE = 'NO' ) 'multiplication constant: '
READ*, eff
WRITE( *, '(A)', ADVANCE = 'NO' ) 'fast fission factor (probability of fast neutrons undergoing fission): '
READ*, e
WRITE( *, '(A)', ADVANCE = 'NO' ) 'moderator to fuel ratio (usually 0.874): '
READ*, p
WRITE( *, '(A)', ADVANCE = 'NO' ) 'Number of atoms of fuel / cm^3: '
READ*, N1
WRITE( *, '(A)', ADVANCE = 'NO' ) 'Number of atoms/cm^3 of moderator: '
READ*, N2
WRITE( *, '(A)', ADVANCE = 'NO' ) 'Number of atoms/cm^3 of coolant: '
READ*, N3
WRITE( *, '(A)', ADVANCE = 'NO' ) 'microscopic cross section for the fuel: '
READ*, S1
WRITE( *, '(A)', ADVANCE = 'NO' ) 'microscopic cross section for the moderator: '
READ*, S2
WRITE( *, '(A)', ADVANCE = 'NO' ) 'microscopic cross section for the coolant: '
READ*, S3
f = (N1*S1)/((N1*S1)+(N2*S2)+(N3*S3))
k = f*eff*e*p
PRINT*, 'Neutron Multiplication Factor: ', k
END
```

Figure 3.8: Fortran Program for Calculation of Multiplication factor

3.2.7 Criticality Analysis

If the value of $k > 1$, then the chain reaction will pickup speed and there will be more fission reactionsoccurring per second (Supercritical Condition).

If the value of $k = 1$, then the reaction will continue self sustaining with the same rate (Critical Condition).

If the value of $k < 1$, then the reaction will eventually die out as the number of fission reactions will decrease (Subcritical Condition).

3.3 Heat Flow in Nuclear Reactor

In steady state condition, all the heat generated must be released. This happens by the coolant flow which is either liquid or gaseous. The nature and operation of coolant is different according to the type of reactor.

The temperature in a reactor is varied. Usually every fuel rod is of different temperature with the rod in the center of the reactor being the hotter compared to others. The maximum fuel temperature depends on the reactor power level, cooling system, and nature of fuel. The temperature should not exceed a limit value because with calculations we have to see the fuel rod temperature does not melt the other metal and claddings.

3.3.1 Thermodynamic Considerations

According to thermodynamics, in a reactor energy is in form of heat is generated through the nuclear fuel and transferred to a moving fluid. The coolant absorbs heat from fuel rods which is generated at q watts rate, with inlet and outlet temperature as Tin and Tout respectively, with fluid coolant rate of w lb/hr or kg/hr. Thus heat required is,

$$\int_{in}^{out} c_p\,(T)\,dT \tag{3.14}$$

$c_p(T)$ - Specific heat. With the rate of flow of fluid coolant being w lb/hr or kg/hr the heat produced in the reactor is given by

$$q = w\int_{in}^{out} c_p(T)dT$$

When we write this in terms of thermodynamic function enthalpy

$$h = u + P.v$$

h_{in}, h_{out} being the specific enthalpies of the coolant,

$$h_{out} = h_{in} + w\int_{T_{out}}^{T_{in}} c_p(T)dT$$

Therefore we can see that the above equation (3.14) as can be written again as

$$q = w(h_{out} - h_{in})$$

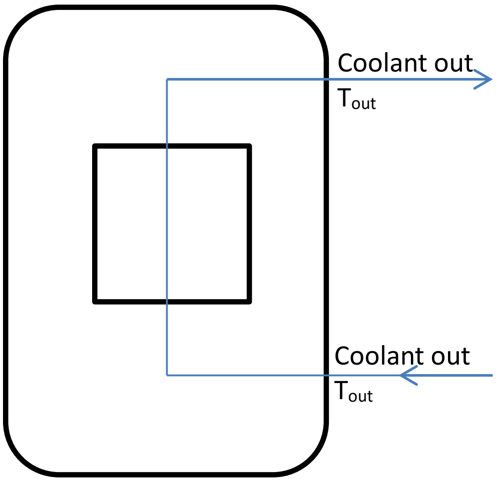

Figure 3.9: Coolant flowing in a reactor

3.3.2 Heat Generation in Reactors

We know that the energy released in a nuclear reactor are– KE of the neutrons, gamma and beta rays. Reactor parts absorb this energy.

3.3.3 Flux Generation

According the flux study we can find the flux for cylindrical fuel element by the Laplacian approximation to cylindrical coordinates and reactor equation becomes,

$$\frac{1}{r}\frac{d}{dr}r\frac{d\varphi}{dr} + B^2\varphi = 0 \tag{3.15}$$

$$\frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} + B^2\varphi = 0 \tag{3.16}$$

Using Boundary condition of $\varphi(R) = 0$.

Equation (3.16) being Bessel equation's special case,

$$\frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} + \left(B^2 - \frac{m^2}{r^2}\right)\varphi = 0 \tag{3.17}$$

In which m is a constant. Equation (3.17) has solutions denoted as $J_m(B.r)$ and $Y_m(B.r)$ and are known as first and second kind, respectively.

Comparing (3.16) and (3.17) shows that, m should be equal to zero. Solution is given by,

$$\varphi = AJ_0(Br) + CY_0(Br) \tag{3.18}$$

The function $J_0(x)$ and $Y_0(x)$ are plotted in the Figure 2. C can be taken as zero since φ is finite in the reactor boundary.

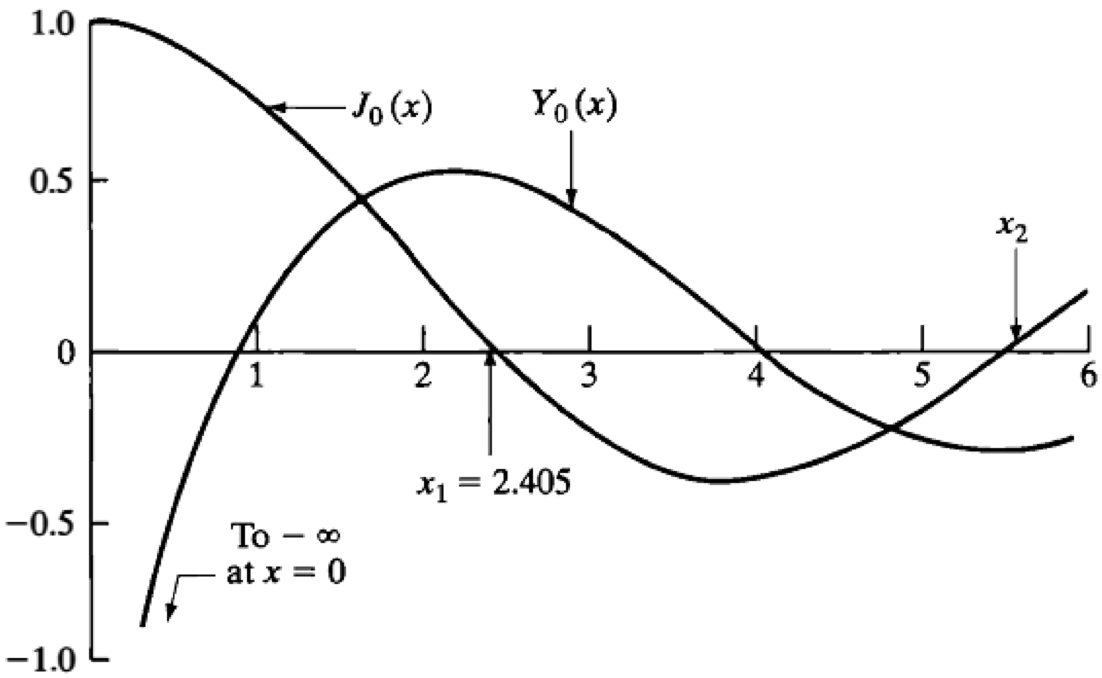

Figure 3.10: The Bessel functions $J_0(x)$ and $Y_0(x)$

Thus φ reduces to,

$$\varphi = AJ_0(Br)$$

At $\tilde{R}$ the value of φ becomes zero

$$\varphi(\tilde{R}) = AJ_0(B.r) = 0$$

As shown in the above figure, the function $J_0(x)$ = 0 at x, marked as x_1, x_2 … so that $J_0(x_n) = 0$. This indicates it is fulfilled on condition that B has a value of

$$B_n = \frac{x_n}{\tilde{R}}$$

Known as Eigen values. For critical reactor only lowest value is required which follow buckling as

$$B_1^2 = \left(\frac{x_1}{\tilde{R}}\right)^2 = \left(\frac{2.405}{\tilde{R}}\right)^2$$

This one-group flux is then

$$\varphi = AJ_0\left(\frac{2.405r}{\tilde{R}}\right)$$

The constant A is again determined by

$$P = \ E_R \textstyle\sum_f \displaystyle\int \varphi(r)dV$$

Infinite cylinder will have $dV = 2\pi r dr$. Thus,

$$P = \ 2\pi E_R \textstyle\sum_f \displaystyle\int_0^R \varphi(r) r \ dr$$

$$P = \ 2\pi E_R \textstyle\sum_f A \displaystyle\int_0^R J_0\left(\frac{2.405r}{\tilde{R}}\right) r \ dr$$

The integral can be evaluated using the formula

$$\int J_0(x').x'.dx' = xJ_1(x)$$

Thus,

$$P = \frac{2\pi E_R \sum_f R^2 A J_1(2.405)}{2.405} = \ 1.35 \ E_R \textstyle\sum_f R^2 A$$

$$A = \ \frac{P}{1.35 \ E_R \sum_f R^2}$$

Therefore the final expression for φ is given by,

$$\varphi = \ \frac{0.738 \ P}{E_R \sum_f R^2} \ J_0 \ \left(\frac{2.405r}{R}\right) \tag{3.19}$$

For a finite cylinder flux be determined by the distance *r* from axis and z from cylinder's midpoint. We take height ***H*** and radius ***R*** for the fuel element. The Laplacian appropriate to cylindrical coordinates, the reactor equation becomes

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \varphi}{\partial r} + \ \frac{\partial^2 \varphi}{\partial z^2} + \ B^2 \varphi = 0$$

After differentiation,

$$\frac{d^2 \varphi}{\partial r^2} \ + \ \frac{1}{r}\frac{\partial \varphi}{\partial r} + \ \frac{\partial^2 \varphi}{\partial z^2} + B^2 \varphi = 0$$

Besides satisfying this equation φ must also fulfil $\varphi\left(\tilde{R},z\right) = 0$ and $\varphi\left(r,\frac{\tilde{H}}{.2}\right) = 0$. Thus we obtain the following solution

$$\varphi(rz) = R(r).Z(z)$$

Upon change into in equation above, one obtains

$$\frac{1}{R}.\frac{1}{r}.\frac{\partial}{\partial r}r.\frac{\partial R}{\partial r} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -B^2$$

Equation must be satisfied for any r and z combination, the start terms will be constants thus we can write,

$$\frac{d^2R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} + {B_r}^2 R = 0$$

This is equivalent to Bessel's equation of the first-kind order zero as before. Further,

$$\frac{d^2Z}{\partial z^2} + {B_z}^2 Z = 0$$

The buckling B^2 is

$$B^2 = {B_r}^2 + {B_z}^2$$

Whose solution is

$$Z(z) = A\, cos\, B_z z + C \sin B_z z$$

Which is same as infinite slab reactor. The solution must satisfy the boundary conditions and be positive definite. Thus we get the solution,

$$\varphi\,(r,z) = AJ_0\left(\frac{2.405.r}{\tilde{R}}\right)\cos(\frac{\pi.z}{\widetilde{H}}) \tag{3.20}$$

Where $\widetilde{H} = H + 2d$ and $\tilde{R} = R + d$. The constant A may be determined as before if the reactor power is known.

Geometry	Dimensions	Buckling	Flux	A	Ω
Infinite slab	Thickness a	$\left(\frac{\pi}{a}\right)^2$	$A\cos\left(\frac{\pi x}{a}\right)$	$1.57P/aE_R\Sigma_f$	1.57
Rectangular parallelepiped	$a \times b \times c$	$\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2+\left(\frac{\pi}{c}\right)^2$	$A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$	$3.87P/VE_R\Sigma_f$	3.88
Infinite cylinder	Radius R	$\left(\frac{2.405}{R}\right)^2$	$AJ_0\left(\frac{2.405r}{R}\right)$	$0.738P/R^2E_R\Sigma_f$	2.32
Finite cylinder	Radius R Height H	$\left(\frac{2.405}{R}\right)^2+\left(\frac{\pi}{H}\right)^2$	$AJ_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$	$3.63P/VE_R\Sigma_f$	3.64
Sphere	Radius R	$\left(\frac{\pi}{R}\right)^2$	$A\frac{1}{r}\sin\left(\frac{\pi r}{R}\right)$	$P/4R^2E_R\Sigma_f$	3.29

Figure 3.11: Buckling and flux value for a critical bare reactor (Assumption taken is d is small)

3.3.4 Heat Production in Fuel Elements (Fuel Rods)

If E_d is the energy deposited, the rate of heat production at the point **r** is given by

$$q'''(\boldsymbol{r}) = E_d \int_0^\infty \sum\nolimits_{fr}(E)\varphi(\boldsymbol{r},E)dE$$

Where $\sum_{fr}(E)$the macroscopic fission is of the fuel and $\varphi(\boldsymbol{r},E)$ is the energy-dependent flux. The unit for this expression is MeV/sec-cm^3.

The thermal flux is then,

$$\varphi(r,z) = \frac{3.63P}{E_R\overline{\sum}_f V}J_0\left(\frac{2.405r}{R}\right)cos\frac{\pi z}{\widetilde{H}} \qquad (3.21)$$

Where,

P = total power

E_R = recoverable energy,

V = reactor volume, and

$\tilde{R}$ and $\widetilde{H}$ are the extrapolated boundaries.

$\overline{\sum_f}$ = Macroscopic fission cross-section

$\overline{\sum_{fr}}$ the total fission cross-section. In the entire core is $\overline{\sum_{fr}}\,.x.n\pi a^2H$. Average value of $\overline{\sum_f}$ in the core is

$$\overline{\sum}_f = \frac{\overline{\sum}_{fr} n\pi a^2 H}{\pi R^2 H} = \frac{\overline{\sum}_{fr} n a^2}{R^2}$$

And the flux is,

$$\varphi_T(r,z) = \frac{3.63\ P}{E_R \overline{\sum}_{fr} V a^2 n} J_0 \left(\frac{2.405 r}{R}\right) cos \frac{\pi z}{\widetilde{H}} \quad where\ V = \pi R^2 H \qquad (3.22)$$

Therefore,

$$\varphi_T(r,z) = \frac{1.16\ P}{E_R \overline{\sum}_{fr} H a^2 n} J_0 \left(\frac{2.405 r}{R}\right) cos \frac{\pi z}{\widetilde{H}} \qquad (3.23)$$

Thus the rate of heat production becomes,

$$q'''(r,z) = \frac{1.16\ P\ E_d}{E_R H a^2 n} J_0 \left(\frac{2.405. r}{R}\right) cos \frac{\pi z}{\widetilde{H}} \qquad (3.24)$$

From the above equation maximum rate of heat production occurs at the centre. Therefore maximum q''' can be given by,

$$q'''_{max} = \frac{1.16\ P\ E_d}{E_R H a^2 n} \qquad (3.25)$$

The maximum rate is

$$q'''_{max}(r) = q'''_{max}\, J \left(\frac{2.405\ r}{\tilde{R}}\right) \qquad (3.26)$$

The total rate is given by,

$$q_r(r) = \pi a^2 \int_{-H/2}^{H/2} q'''(rz)\, dz \qquad (3.27)$$

$q'''(rz)$ From above equation gives,

$$q_r(r) = \frac{1.16\ P\ E_d}{E_R H n} J_0 \left(\frac{2.405 r}{R}\right) \int_{-H/2}^{H/2} \cos\left(\frac{\pi z}{H}\right) dz = \frac{2.32\ P\ E_d}{E_R n} J_0 \left(\frac{2.405 r}{R}\right) \qquad (3.28)$$

The value of q'''_{max} is overestimated if we consider a real reactor with non-uniform or reflected fueled reactor. To this effect we see the following expression,

$$q'''_{max} = E_d \overline{\sum}_{fr} \varphi_{max}, \qquad (3.29)$$

Where φ_{max} is the maximum value. Total reactor power is

$$P = E_R \overline{\sum}_f \varphi_{av} V, \tag{3.30}$$

Above equations (3.29) and (3.340) and rearranging gives

$$q'''_{max} = \frac{PE_d \overline{\sum}_{fr} \varphi_{max}}{E_R \overline{\sum}_f \varphi_{av} V} = \frac{PE_d \overline{\sum}_{fr} \Omega}{E_R \overline{\sum}_f V} \tag{3.31}$$

Where Ω is the maximum to average flux ratio. Equation for $\overline{\sum}_f$ yields

$$q'''_{max} = \frac{P.E_d.R^2.\Omega}{a^2.n.V.E_R} = \frac{PE_d\Omega}{\pi H a^2 n E_R} \tag{3.32}$$

For an actual Reactor (Triga Mark II) we can take $\Omega = 2.4$. The comparing Equation (3.22) and (3.32) shows that

$$(q'''_{max})_{actual} = \frac{\Omega}{1.16\pi}(q'''_{max})_{bare} \cong \frac{2}{3}\left(q'''_{max}\right)_{bare} \tag{3.34}$$

3.3.5 Heat Flow by Conduction

Energy from a reactor is removed using two heat transfer process – *conduction* and *convection*. There is no macroscopic motion of any part of the body in conduction. It is only due to the difference in temperature between the bodies. Heat convection includes the heat transfer through fluid (here the coolant). It is also due to temperature change but here there is macroscopic movement of the body during heat transfer. Thus the heat is transferred to the coolant (water) from the fuel surface using conduction and then out of the system using convection in the fluid medium.

3.3.6 The Equations of Heat Conduction

Heat conduction obeys *Fourier's Law*,

$$q'' = -k.grad(T)$$

Where q'', is the heat flux, which equals the rate f that is the outward normal heat flow. k is the thermal conductivity whose value is defined for all the substances. The function T is of temperature. We can see the similarity between this and Fick's law that we used for neutron density calculations.

Let us consider an arbitrary volume V of a material where heat is being produced. We can say that from conservation of energy, during steady state the rate of heat that flows out of the surface V. This can be written as

$$\begin{bmatrix} Net\ heat\ flow\ out \\ of\ the\ body \end{bmatrix} - \begin{bmatrix} heat\ produced \\ within\ V \end{bmatrix} = 0$$

Thus solving the above we get,

$$Heat - flow = \int_A q''n\,.dA$$

Using divergence theorem the equation (46) becomes,

$$Heat - flow = \int_V div\, q''\, dV$$

Heat produced within V is equal to

$$Heat - flow = \int_V q'''.dV$$

Thus the relationship is obtained

$$div\,(q'') - q''' = 0 \qquad (3.35)$$

This result is the *steady state equation of conductivity* for heat transfer and is analogous to the equation of continuity in neutron flux generation. There is no term in the equation (3.35) equivalent to the absorption term does not vanish in medium same that neutrons do.

Thus rearranging the above equations and dividing it by k gives,

$$\nabla^2 T = \frac{q'''}{k} = 0 \qquad (3.36)$$

This is known as *Poisson's equation*. For no heat generation area, $q''' = 0$ and equation (51) reduces to

$$\nabla^2 T = 0 \qquad (3.37)$$

This is known as the *Laplace's equation*.

Now using these concepts we will be able to calculate the heat from a fuel rod for the max-temperature in the fuel. It is important to nte that the maximum temperature should not exceed the safety limit.

3.3.6.1 Plate-Type Fuel Elements

Consider a plate-type fuel-element where the fuel central strip – "Meat" has a thickness of *2a* and cladding with thickness of *b*.

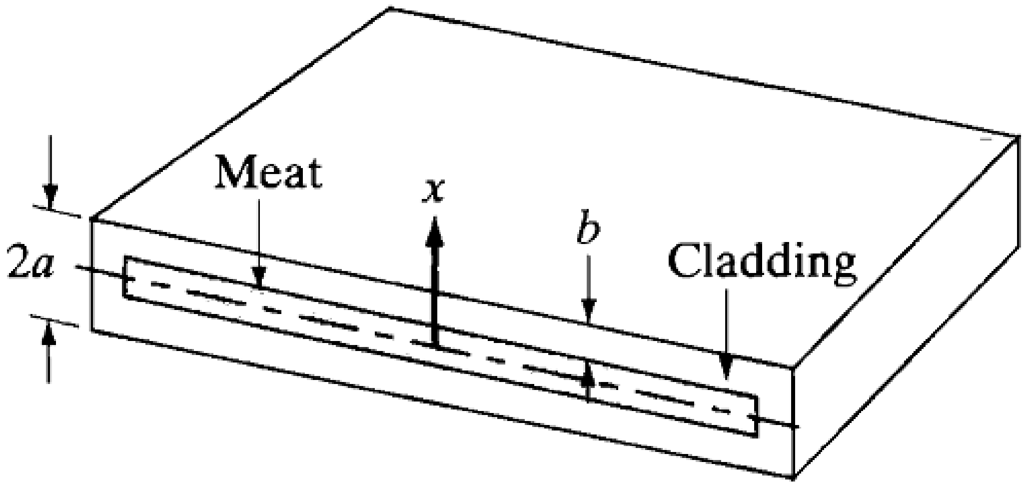

Figure 3.12: Plate Type Fuel Element

While doing the next calculations we will take few important assumptions that

- Heat generated uniformly at rate of q'''.
- Temperature reached steady state with distribution throughout the element.
- Thickness of the element is negligible compared to the width or length of the element, therefore negligible amount of heat flows through the edges.
- Heat only flows in the 'x' direction.

The temperature profile now can be calculated using,

$$\frac{d^2(T)}{d(x^2)} + \frac{q'''}{k_f} = 0$$

Where k_f- thermal conductivity.

Boundary conditions used are,

$$T(0) = T_m$$

T_m-maximum (ventral temperature) at $x = 0$.

$$\frac{d(T)}{dx} = 0$$

Integrating Equation twice gives the following general solution

$$T = \frac{q'''}{2k_f}(x)^2 = C_1.(x) + C_2$$

Using the boundary conditions we get,

$$T = T_m - \frac{q'''}{2k_f}x^2$$

Thus surface temperature will be,

$$T_s = T_m - \frac{q'''.a^2}{2k_f} \tag{3.38}$$

If the area of one face of the fuel element is *A*, then the volume of the fuel element is *2aA*. Therefore the rate of heat production

$$q = q'''(2aA)$$

Heat for one face can be given by,

$$q = q'''(a.A)$$

By Fourier's Law,

$$q'' = -k_f\frac{dT}{dx}$$

Heat flow per unit area $\quad q'' = (q'''.a)$

Total heat rate can then be given by,

$$q = q'''(aA) = q''A \tag{3.39}$$

Rearranging above equations we can get,

$$q''' = \frac{2k_f(T_m - T_s)}{a^2} \tag{3.40}$$

Using (3.39) in (3.40) we get,

$$q = \frac{2k_f(T_m - T_s)A}{a} \tag{3.41}$$

Equation (3.41) can be rewritten as,

$$q = \frac{(T_m - T_s)}{\left[\frac{a}{2k_f A}\right]} \tag{3.42}$$

We can see the analogous of the above equation to current and voltage where temperature difference is like the potential difference, heat is equivalent to current and denominator is thermal resistance.

Now let us see the temperature distribution in the cladding. There is negligible generation of heat in the cladding. Therefore,

$$\frac{d^2T}{dx^2} = 0$$

Boundary conditions for the cladding is

$$T(a) = T_s,$$

$$T(a + b) = T_c\ (outer\ surface\ of\ the\ cladding)$$

The general solution for the above equation (64) is

$$T = C_1 x + C_2$$

Substituting the boundary conditions to (65)

$$T_s = C_1 a + C_2$$

$$T_c = C_1(a + b) + C_2$$

Solving the above equations to find out the constants,

$$C_1 = \frac{T_c - T_s}{b}$$

$$C_2 = \frac{T_s(a + b) - T_c a}{b}$$

Substituting these values and rearrange to get the cladding temperature profile

$$T = T_s - \frac{(x - a)(T_s - T_c)}{b} \tag{3.43}$$

The temperature profile inside the fuel region is quadratic whereas in the cladding is linear. From (3.43) we can find $\frac{dT}{dx}$ which is then put to the Fourier's Law with k_c as the conductivity of the cladding.

$$q'' = k_c \frac{T_s - T_c}{b} \tag{3.44}$$

We use $q = q''A$ to find

$$(T_s - T_c) = q\left(\frac{b}{k_c A}\right) \tag{3.45}$$

From above we get

$$(T_m - T_s) = q\left(\frac{a}{2k_f A}\right) \tag{3.46}$$

Adding (3.45) and (3.46),

$$(T_m - T_c) = q\left(\frac{b}{k_c A} + \frac{a}{2k_f A}\right) \tag{3.47}$$

Where $\left(\frac{b}{k_c A} + \frac{a}{2k_f A}\right)$ is the total resistance of the element system.

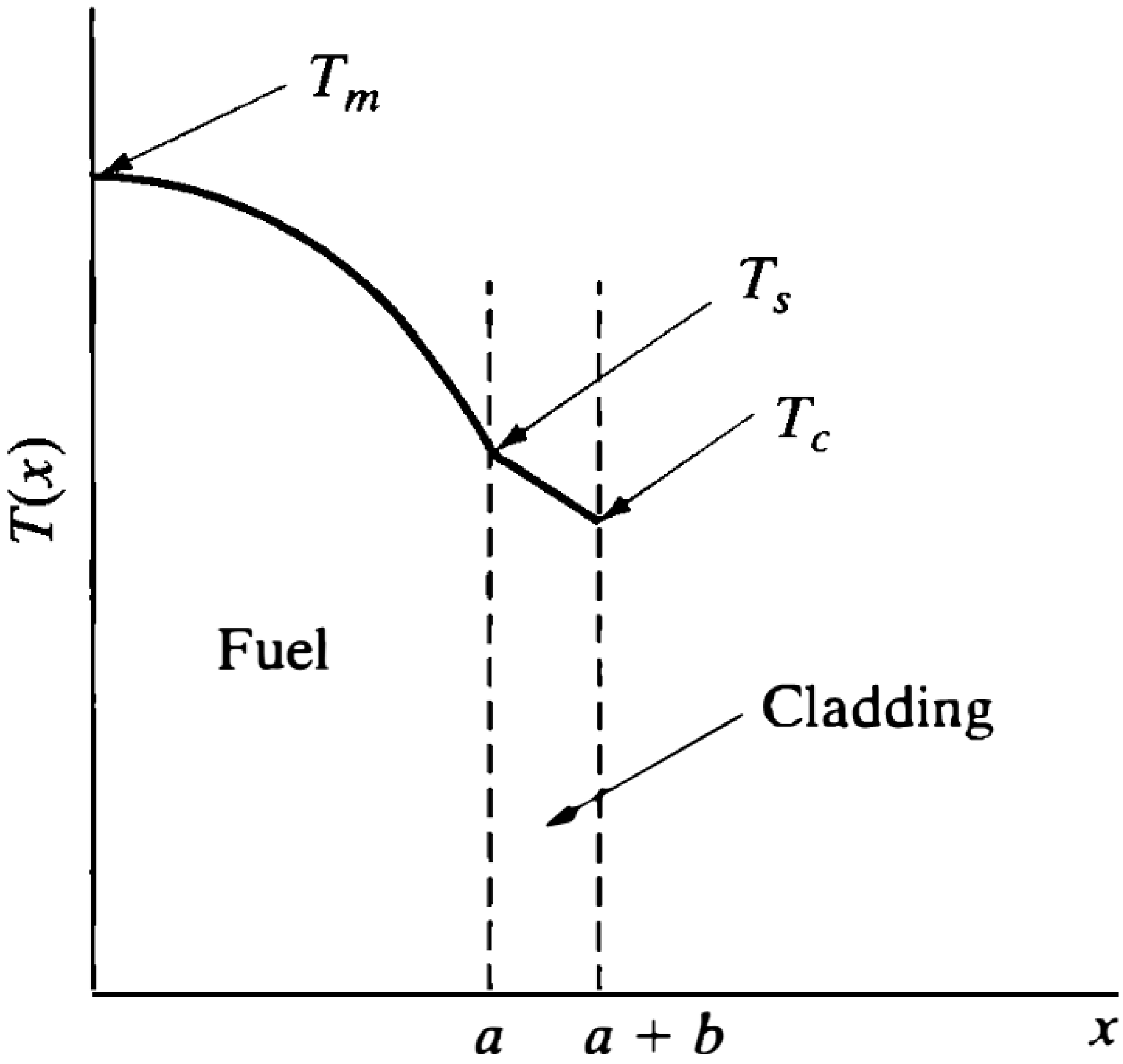

Figure 3.13: Temperature distribution across plate-type fuel-element

3.3.6.2 Cylindrical Fuel Rod

Consider the figure below where inner cylinder represents the fuel rod with a radius of 'a' and the outer cylinder acting as a thickness is the cladding of 'b' that is a radius of 'a+b' from the center. Heat is produced at constant rate as discussed earlier.

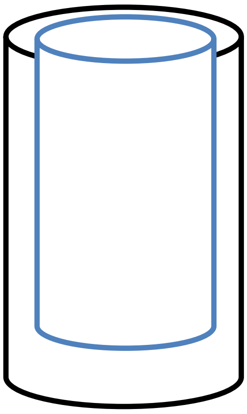

Figure 3.14: Cylindrical Fuel Element

For cylindrical heat flow the following equation is used,

$$\frac{d^2(T)}{d(r^2)} + \frac{1}{r}\frac{d(T)}{d(r)} + \frac{(q''')}{k_f} = 0$$

Boundary conditions are,

(i) Temperature is varying inside the rod

(ii) Temperature at the center is maximum

(iii) $\frac{dT}{dr}$ at $r = 0$ is 0

Using general solution,

$$T = -\frac{q'''.r^2}{4k_f} + C_1 ln\, r + C_2$$

Since $\frac{dT}{dr}$ at $r = 0$ is 0, therefore $\boldsymbol{C_1}$= 0**.** Substituting the value of $\boldsymbol{C_1}$ in above we get,

$$T = -\frac{q'''r^2}{4k_f} + C_2$$

Now using the condition at $r = 0, T(0) = T_m$: Therefore$C_2 = T_m$. Thus,

$$T = T_m - \frac{q'''.r^2}{4.k_f} \tag{3.48}$$

$$q = (Volume)\ x\ q''' = \pi a^2 H q''' \tag{3.49}$$

Where H is the total height of core rods.

For (q''') from equation (3.48) we get,

$$q''' = \frac{T_m - T_s}{\left(\frac{r^2}{4k_f}\right)} \tag{3.50}$$

Now for surface of the fuel rod at $r = a$ we get,

$$q''' = \frac{T_m - T_s}{\left(\frac{a^2}{4k_f}\right)} \tag{3.51}$$

From the relation $q = q'''(Volume)$ we get,

$$q = \pi a^2 H\ .\ \frac{T_m - T_s}{\left(\frac{a^2}{4k_f}\right)}$$

So solving this we get,

$$q = \frac{T_m - T_s}{\left(\frac{1}{4\pi H k_f}\right)} \tag{3.52}$$

The thermal resistance fuel is clearly

$$R_f = \frac{1}{4\pi H k_f}$$

Similarly now for the cladding,

$$\frac{d^2(T)}{d(r^2)} + \frac{1}{r}\frac{d(T)}{d(r)} = 0$$

Thus,

$$T = C_1 \ln r + C_2$$

We know that $T(a) = T_s \; and \; T(a+b) = T_c$

And to find the constants,

$$T = \frac{T_s \,. \ln(a+b) - T_c . \, ln \; a - (T_s - T_c) \; \ln r}{\ln(1 + \frac{b}{a})} \tag{3.53}$$

Cladding temperature thus can be found by,

$$q = -2\pi . \, (a+b) . \, H . \, k_c . \frac{dT}{dr}$$

We also find from differentiation of equation (3.53)

$$q = \frac{2\pi H k_c (T_s - T_c)}{\ln(1 + \frac{b}{a})} \tag{3.54}$$

The cladding resistance thus can be given by,

$$R_c = \frac{\ln(1 + \frac{b}{a})}{2\pi H k_c} \tag{3.55}$$

b<<a and in this case,

$$ln\left(1 + \frac{b}{a}\right) \cong \frac{b}{a}$$

Now equation (3.55) can be written as

$$R_c = \frac{\mathrm{b}}{2\pi a H k_c} \tag{3.56}$$

The heat flowing from the rod outside is,

$$q = \frac{(T_m - T_c)}{(R_f + R_c)} \tag{3.57}$$

3.3.7 Space-Dependent Heat Sources

From previous calculations we know

$$q''' = q'''_{max} \cos\left(\frac{\pi z}{\widetilde{H}}\right)$$

Where z = measure from the mid-point of the rod. A non-uniform heat distribution like the above equation gives a non-uniform temperature distribution.

Since rate of heat generation is

$$q = \pi a^2 H q'''$$

Heat flux,

$$q''(z) = \frac{q}{surface\ area\ of\ rod} = \frac{\pi a^2 H q'''}{2\pi(a+b)H} = \frac{a^2 q'''(z)}{2(a+b)} \tag{3.58}$$

By dividing Eq. (3.57) by $2\pi(a+b)H$:

$$q''(z) = \frac{\left(T_m(z) - T_c(z)\right)}{2\pi.(a+b).H.\left(R_f + R_c\right)} \tag{3.59}$$

3.3.8 Heat Transfer to Coolant

By Newton's law of cooling:

$$q'' = h(T_c - T_b)$$

Where

q'' - Flux,

T_c - Temperature of the surface of the solid

T_b - Appropriate reference temperature of the fluid.

h=5000 $Btu/hrft^2$°F for ordinary water

h =8000 $Btu/hrft^2$°F for heavy water.

T_b is taken to be the mixed mean or bulk temperature of the fluid. This is defined by the formula,

$$T_b = \frac{\int \rho c_p v T \, dA_c}{\int \rho c_p v \, dA_c}$$

Thus heat between the solid and liquid face can be given by,

$$q = q''.A = h.A.(T_c - T_b) \tag{3.60}$$

Therefore,

$$q = q''A = \frac{(T_c - T_b)}{\left(\frac{1}{hA}\right)} \tag{3.61}$$

Thus the thermal resistance for convective heat transfer = $1/hA$.

Therefore plate-type fuel element with cladding has a thermal resistance of,

$$R = \frac{a}{2k_f A} + \frac{b}{k_c A} + \frac{1}{hA} \tag{3.62}$$

Therefore the total heat flow can be given by,

$$q = \frac{(T_m - T_b)}{\frac{a}{2k_f A} + \frac{b}{k_c A} + \frac{1}{hA}} \tag{3.63}$$

Similarly for cylinder the thermal resistance will be

$$R = \left(\frac{1}{4\pi H k_f}\right) + \frac{\ln(1 + \frac{b}{a})}{2\pi H k_c} + \frac{1}{hA} \tag{3.64}$$

Where $A = 2.\pi.(a+b).H$ and now if $a \ll b$,

$$R = \left(\frac{1}{4\pi H k_f}\right) + \frac{\mathrm{b}}{2\pi a H k_c} + \frac{1}{hA} \tag{3.65}$$

And we know,

$$q = \frac{T_m - T_b}{R}$$

Again we need to find the heat flux in the z direction which is given by

$$q''(z) = \frac{q}{2\pi(a+b)H} = \frac{q'''.2\pi a^2 H}{2\pi(a+b)H}$$

Therefore

$$q''(z) = \frac{q''' a^2}{(a+b)} \tag{3.66}$$

And we know,

$$q = q'''.2\pi a^2 H = \frac{T_m - T_b}{R} \tag{3.67}$$

So,

$$q''' = \frac{T_m - T_b}{R.2\pi a^2 H} \tag{3.68}$$

Now substituting this in (3.66)

$$q''(z) = \frac{(T_m(z) - T_b(z))}{2.\pi.(a+b).H.R} \tag{3.70}$$

Considering the cladding temperature,

$$q''(z) = \frac{T_c(z) - T_b(z)}{2\pi(a+b)HR_h} = h[T_c(z) - T_b(z)] \tag{3.71}$$

CHAPTER 4

RESULTS

4.1 2-D Geometry Design

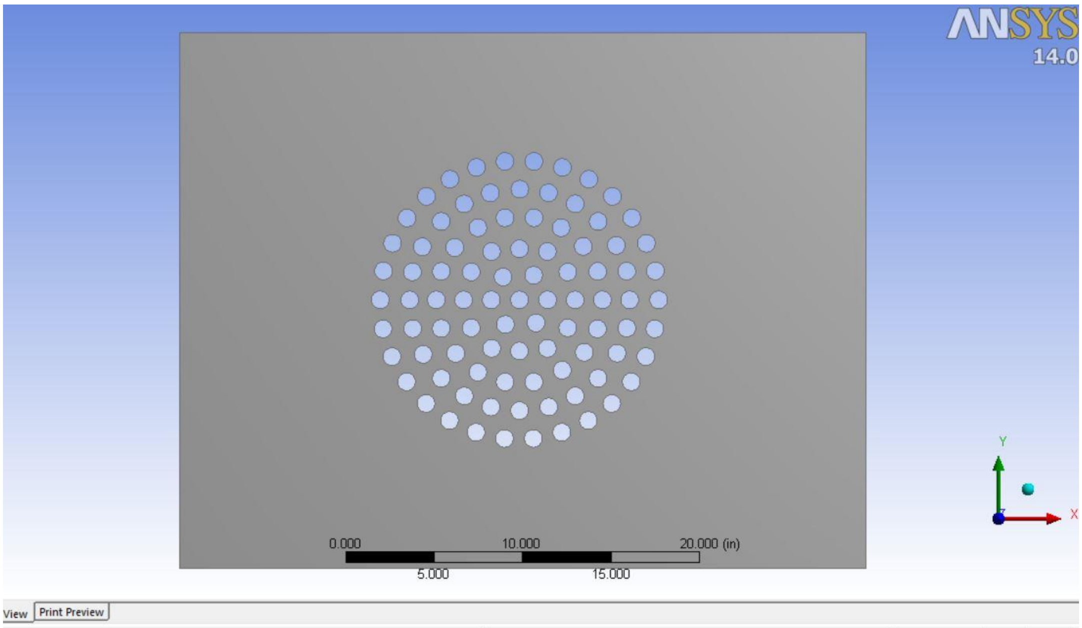

Figure 4.1: 2-D Geometry for the section where water passes

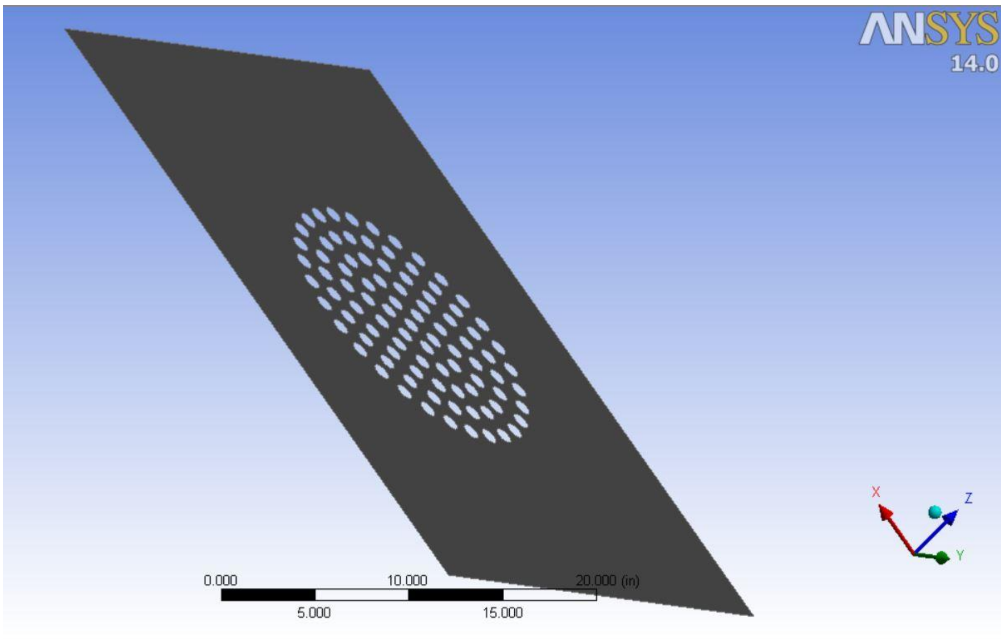

Figure 4.2: Side View to see the 2-D thickness

4.1.1 Working Fluid Properties

We have taken water as the working fluid. The basic properties of the working fluid are:

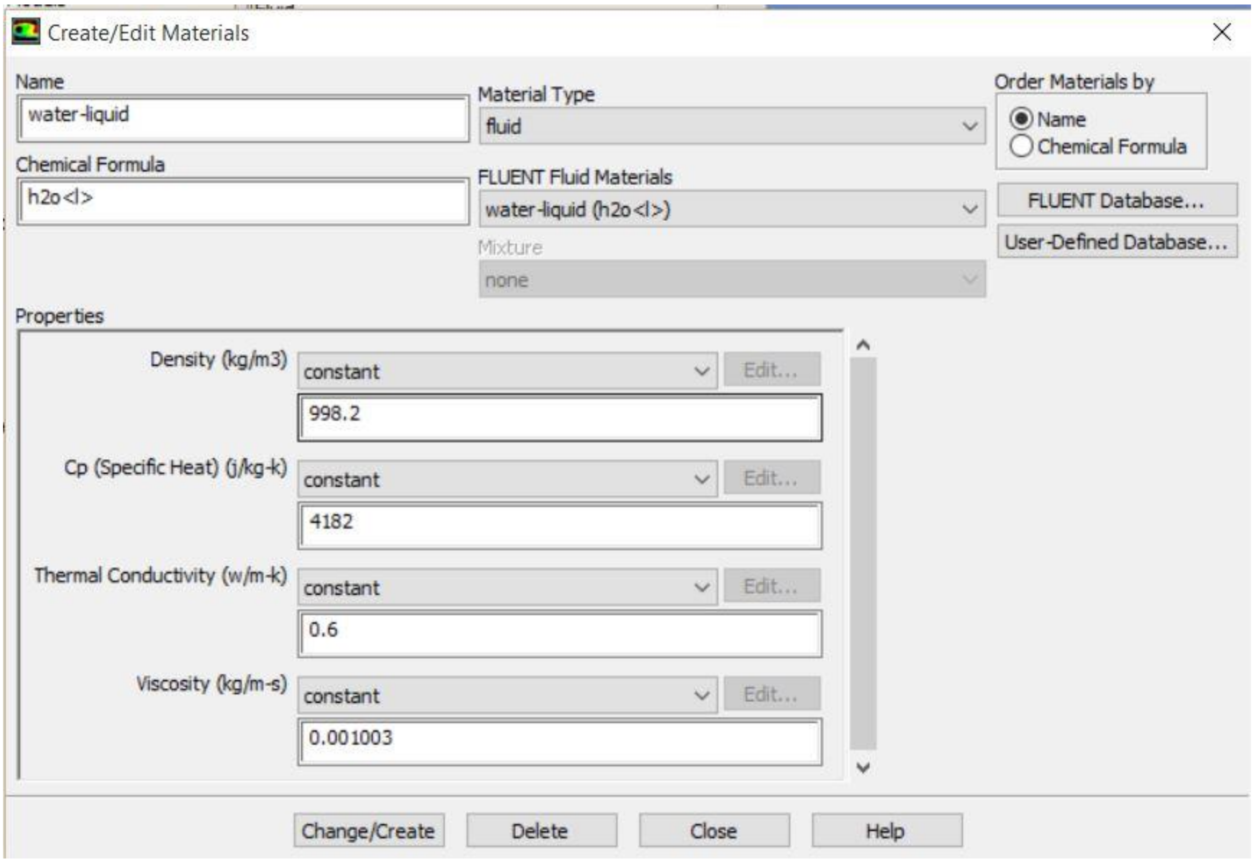

Figure 4.3: Working fluid properties

The flow velocity of the fluid at the inlet wall is 0.9 m/s.

4.2 Meshing

4.2.1 Meshing Details

Details of "Mesh"

Defaults	
Physics Preference	CFD
Solver Preference	Fluent
Relevance	0
Sizing	
Use Advanced Size Fun...	On: Curvature
Relevance Center	Fine
Initial Size Seed	Active Assembly
Smoothing	High
Span Angle Center	Fine
Curvature Normal A...	Default (18.0 °)
Min Size	Default (1.8229e-004 m)
Max Face Size	Default (1.8229e-002 m)
Max Size	Default (3.6457e-002 m)
Growth Rate	Default (1.20)
Minimum Edge Length	7.9796e-002 m
Inflation	
Use Automatic Inflation	None
Inflation Option	Smooth Transition
Transition Ratio	0.272
Maximum Layers	2
Growth Rate	1.2
Inflation Algorithm	Pre
View Advanced Options	No
Assembly Meshing	
Method	None
Patch Conforming Options	
Triangle Surface Mesher	Program Controlled
Advanced	
Shape Checking	CFD
Element Midside Nodes	Dropped
Number of Retries	0
Extra Retries For Assem...	Yes
Rigid Body Behavior	Dimensionally Reduced
Mesh Morphing	Disabled
Defeaturing	
Use Sheet Thickness fo...	No
Pinch Tolerance	Default (1.6406e-004 m)
Generate Pinch on Ref...	No
Sheet Loop Removal	No
Automatic Mesh Based...	On
Defeaturing Toleran...	Default (9.1143e-005 m)
Statistics	
Nodes	60725
Elements	114158
Mesh Metric	None

Figure 4.4: Meshing Details

4.2.2 Meshing Figures

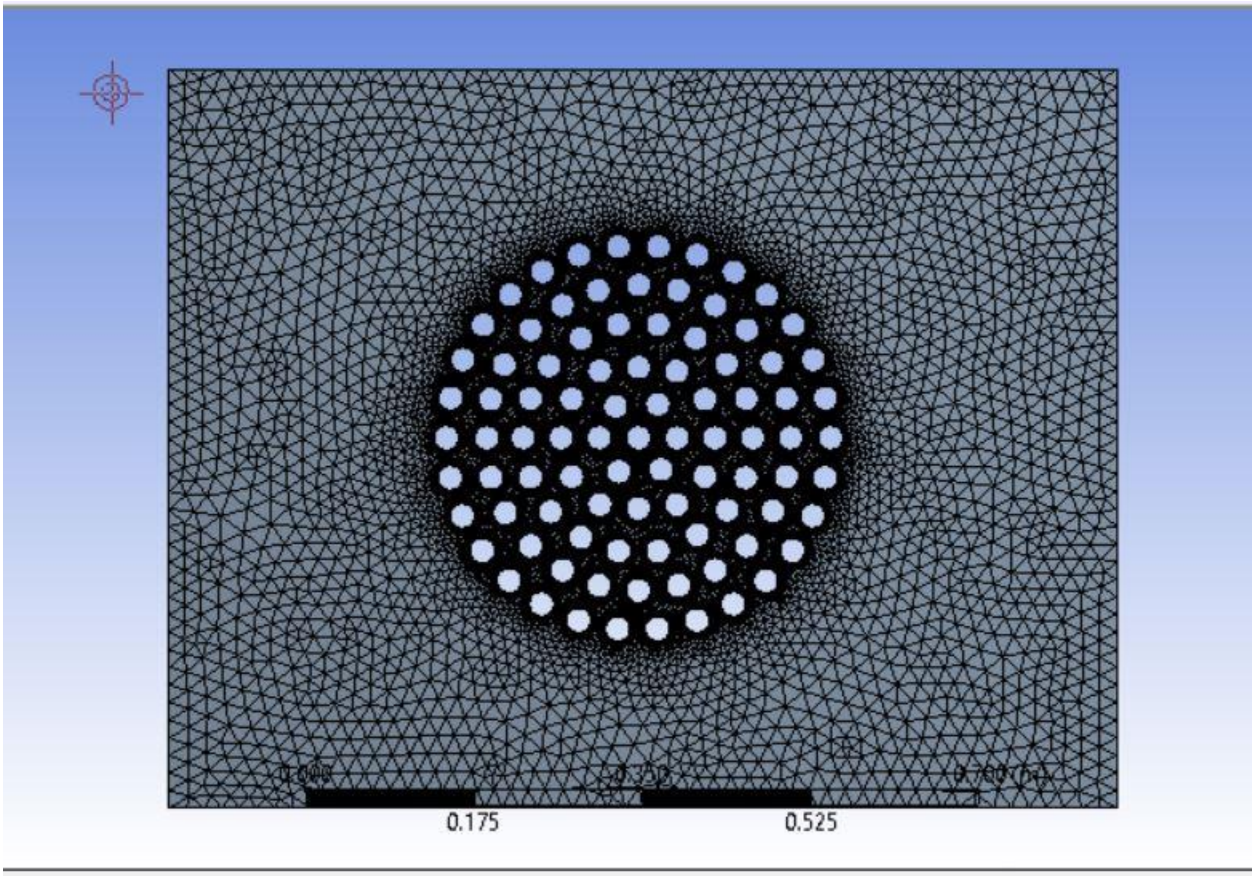

Figure 4.5: Meshing section

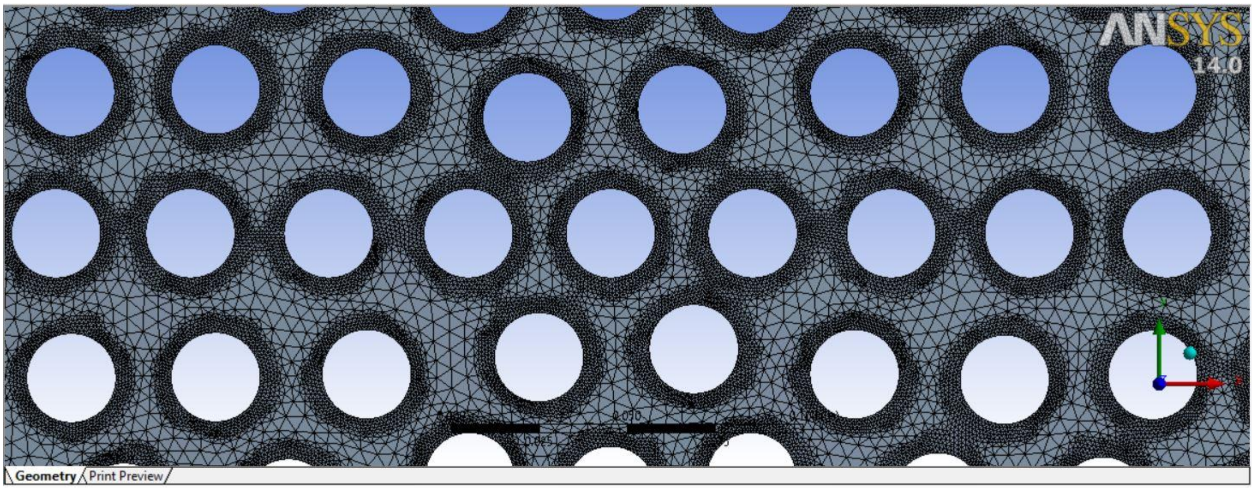

Figure 4.6: Zoomed Meshing

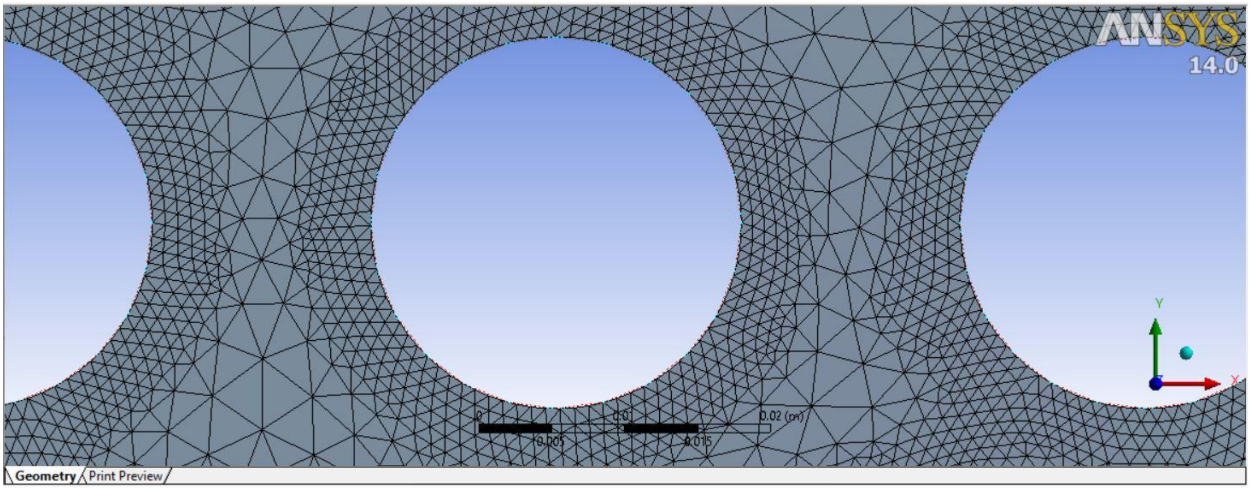

Figure 4.7: Detailed Meshing near the cylinder wall

Meshing is very important to get accurate results. Thus in the part of the heat transfer from the cylindrical wall to the flowing water it is important to be precise. We used high smoothing fine structure to get better results. A total of 114158 elements were created with 60725 nodes.

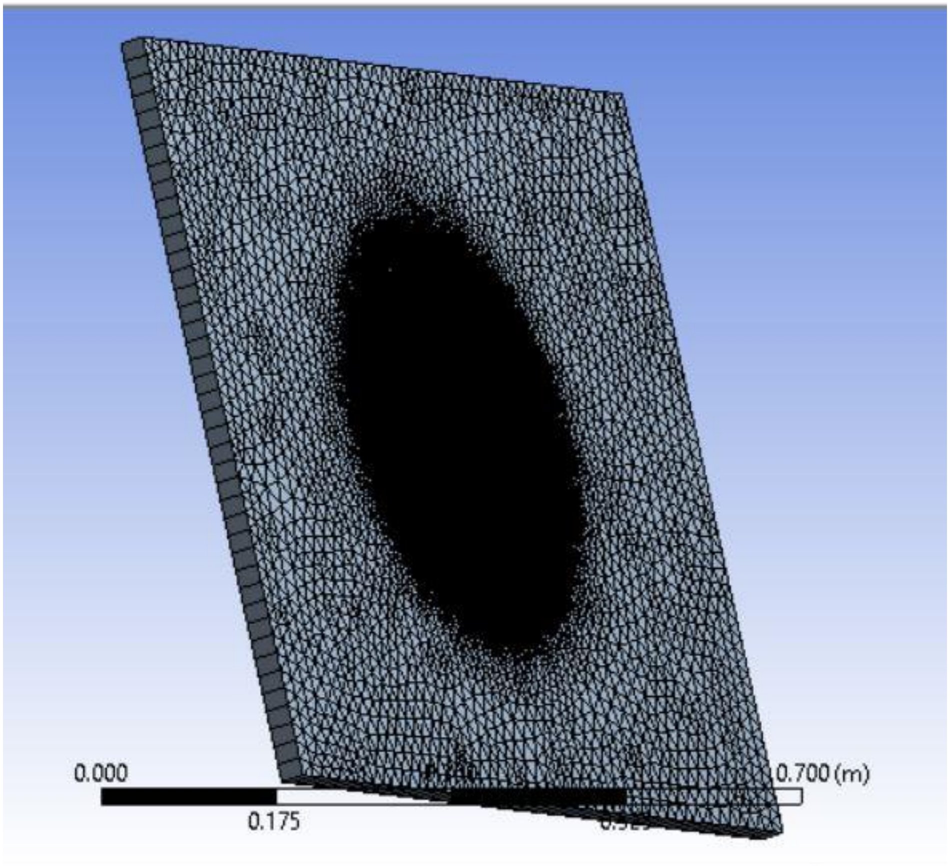

Figure 4.8: Simple Meshing at the inlet wall

4.3 Iterations

We studied the model for 1000 iterations to get a steady state result.

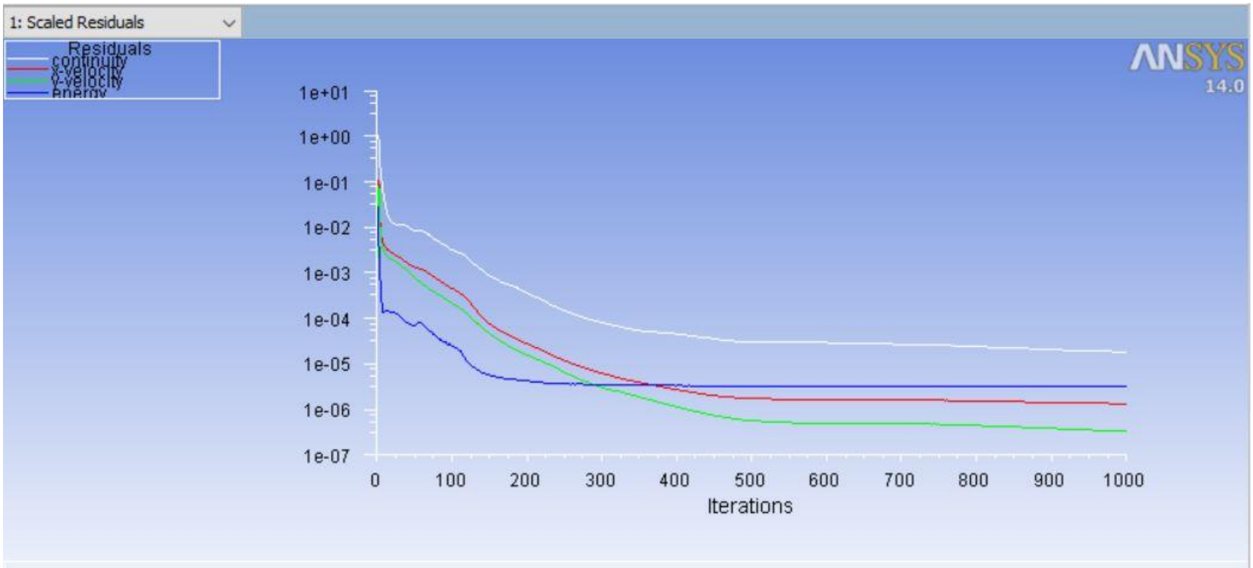

Figure 4.9: Iterations

4.4 Static Pressure

4.4.1 Static Pressure on Cylinder Walls

We see the static pressure on the various cylinder walls and the results are as follows. This is

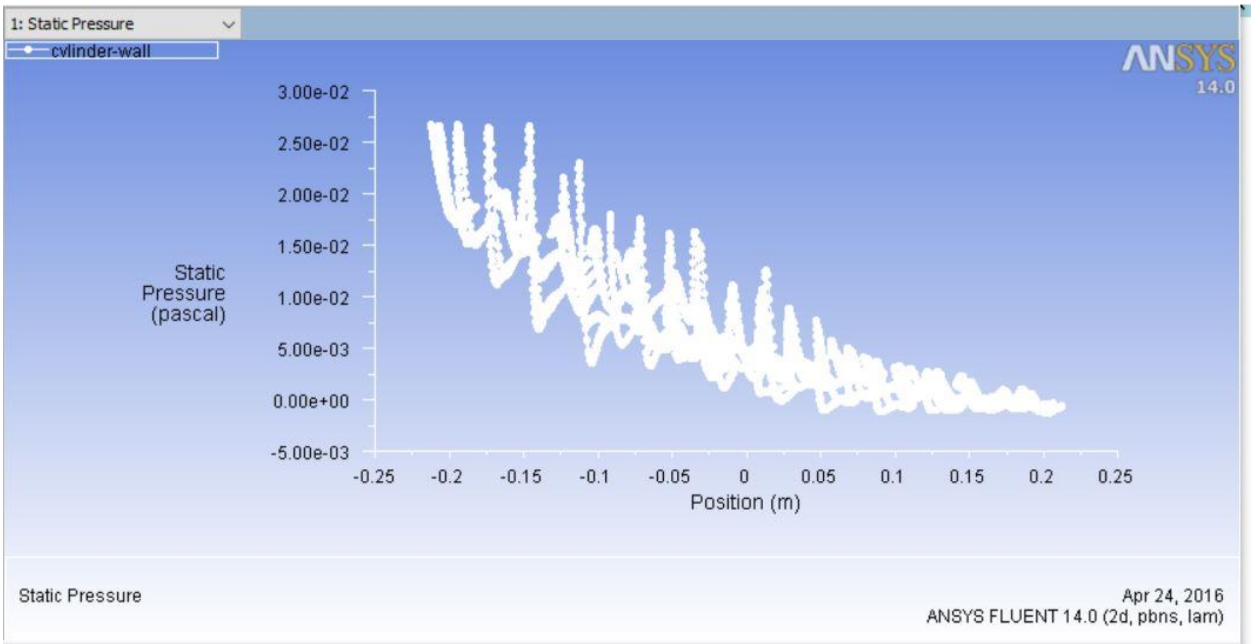

Figure 4.10: Static Pressure on Cylinder Walls

a cumulative graph for all the walls and thus we see many results superimposed.

4.4.2 Static Pressure on Boundary walls

As the velocity of the flow increase after the flow through the cylinder, the pressure has to drop. This is seen perfectly in the following figure.

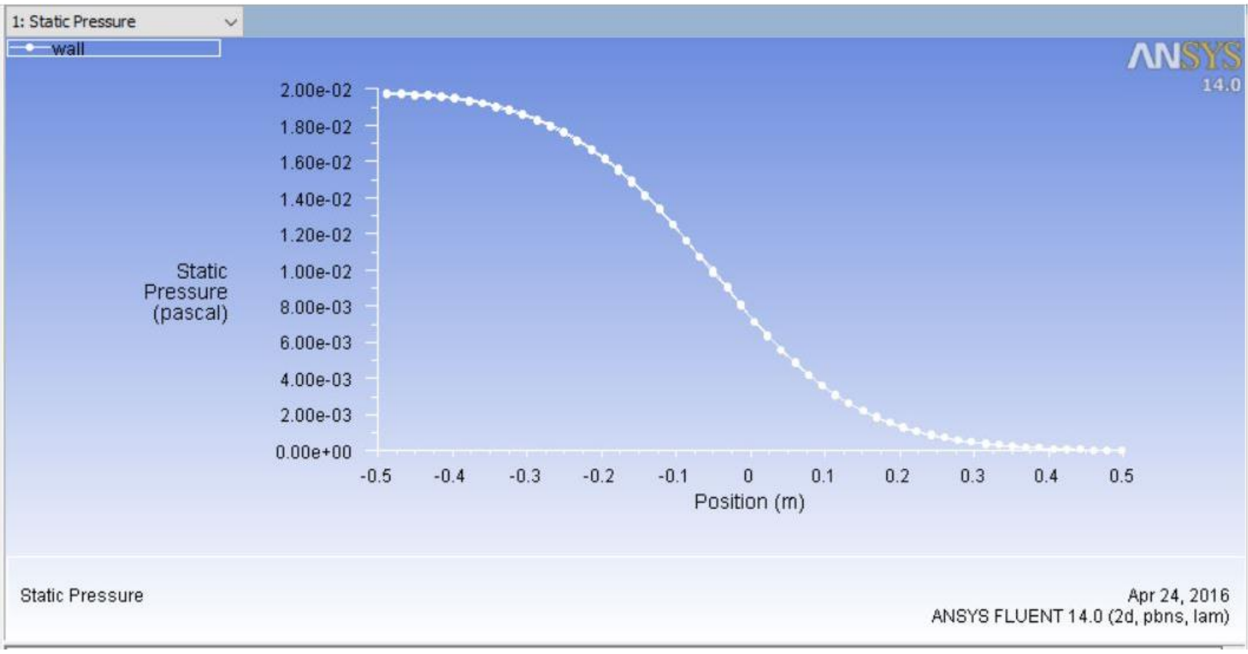

Figure 4.11: Static Pressure on Boundary walls

4.4.3 Static Pressure of the Flow

Since the overall body can be perceived as a cylinder, we see that the pressure drops from the center of the body towards the end. This is because the velocity of the flow increases as we move towards the outlet wall.

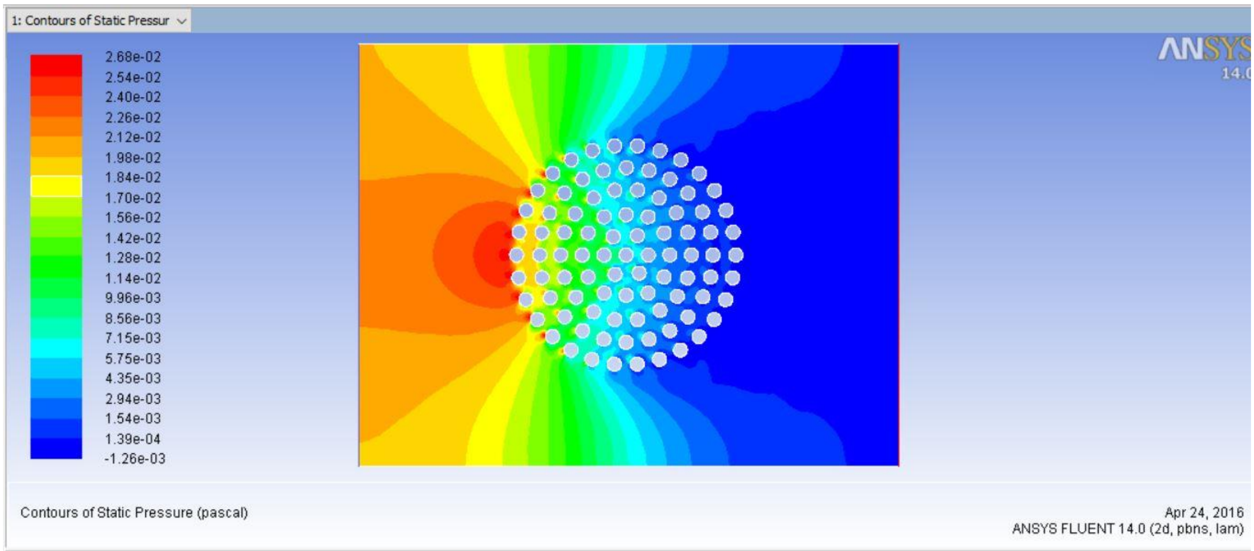

Figure 4.12: Static Pressure of Flow (Contour)

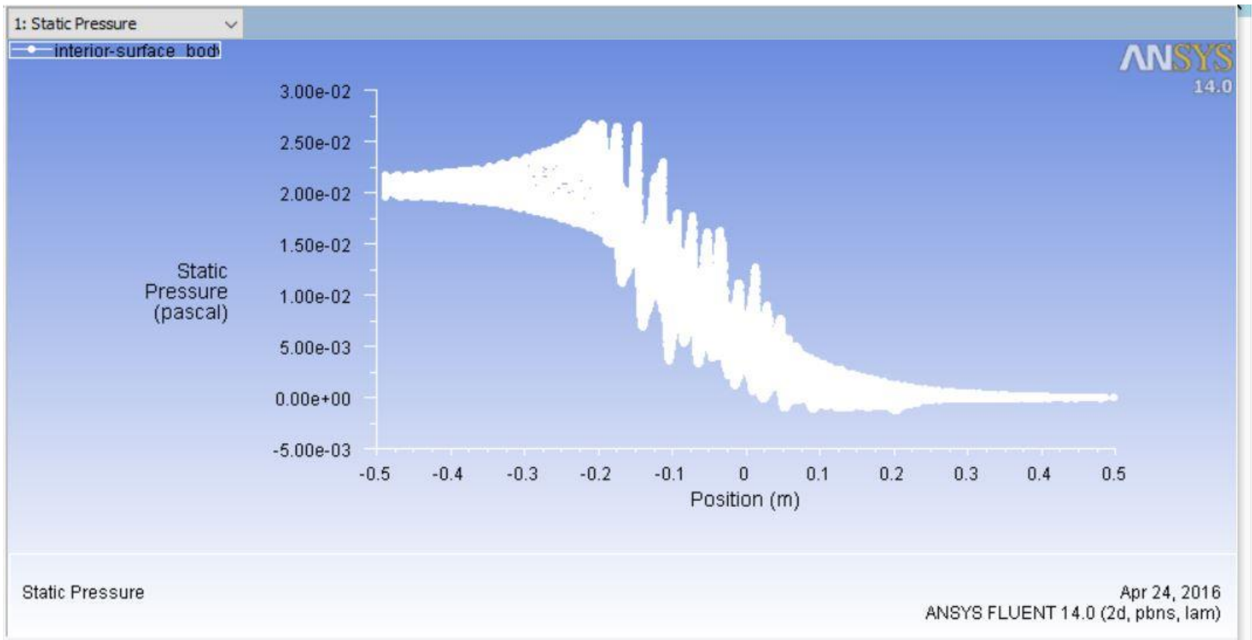

Figure 4.13: Static Pressure of Flow Plot

4.5 Variation in Velocity

As discussed earlier there is change in velocity as it flows through the cylinder walls and it can be seen from the following vector and contour plots.

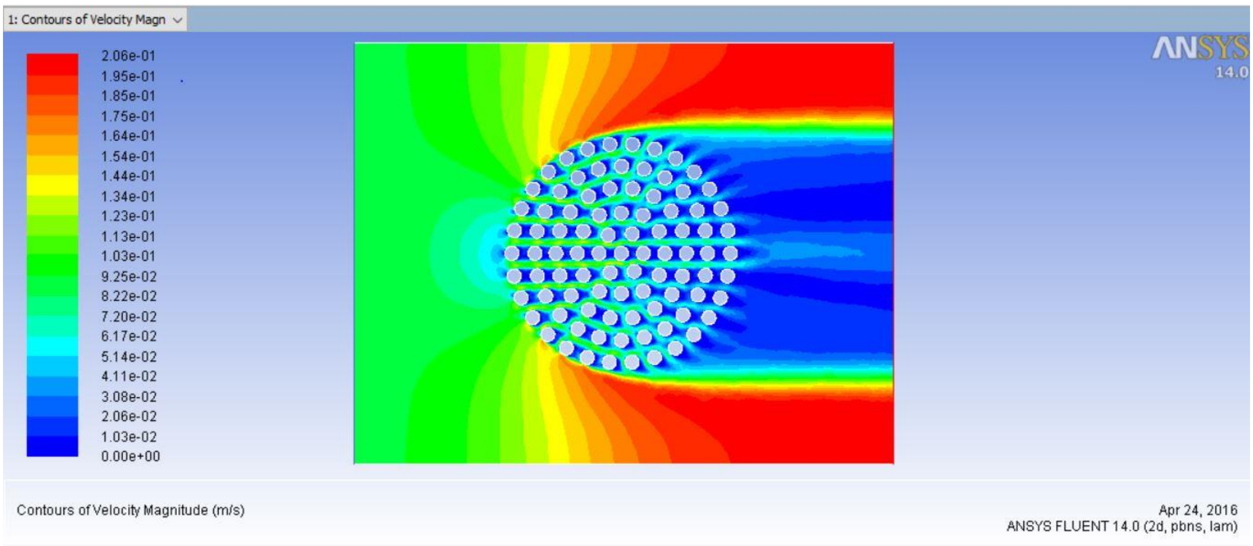

Figure 4.14: Contours of Velocity Magnitude

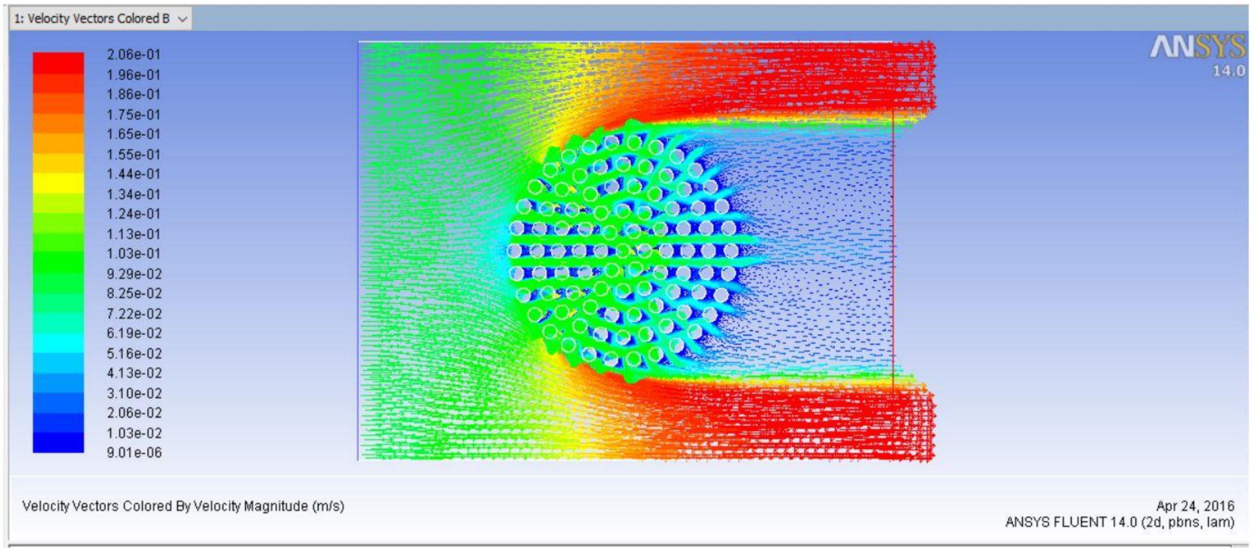

Figure 4.15: Velocity Vectors

There is vortex formation which can be seen in the velocity vector when we take a closer look at it. This is due to the flow of fluid across the cylinder.

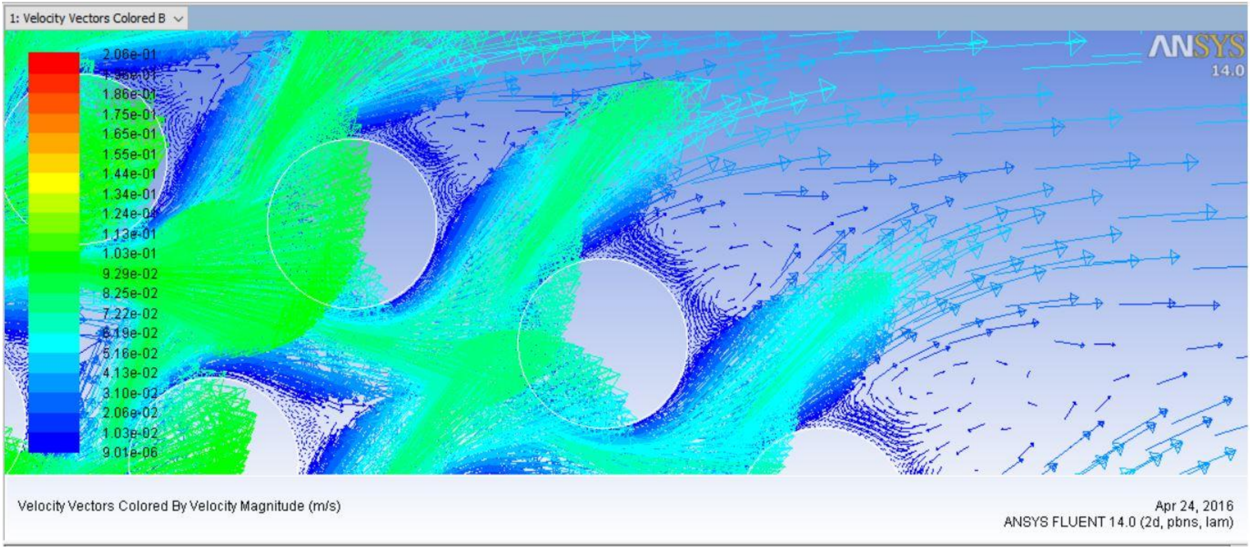

Figure 4.16: Velocity Vortex formation

4.6 Static Temperature

We take the surface temperature on the cylindrical fuel elements after the cladding to be 800K and the fluid entering at 293K and see the flowing change in the static temperature.

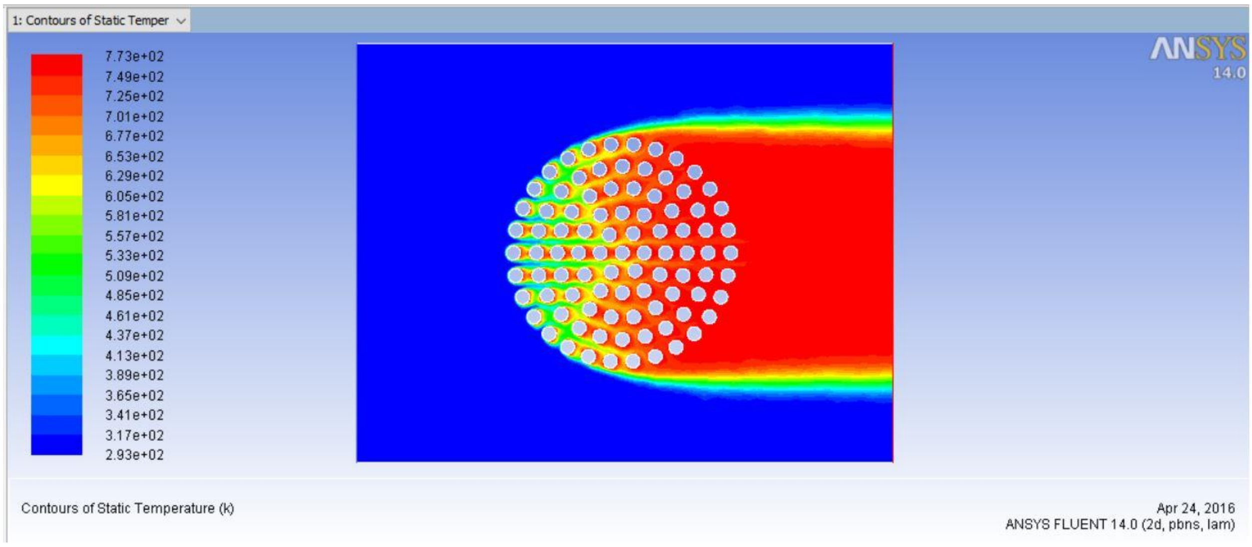

Figure 4.17: Contours of Static Temperature

The surface temperature of the cladded cylindrical fuel element is seen from the following image.

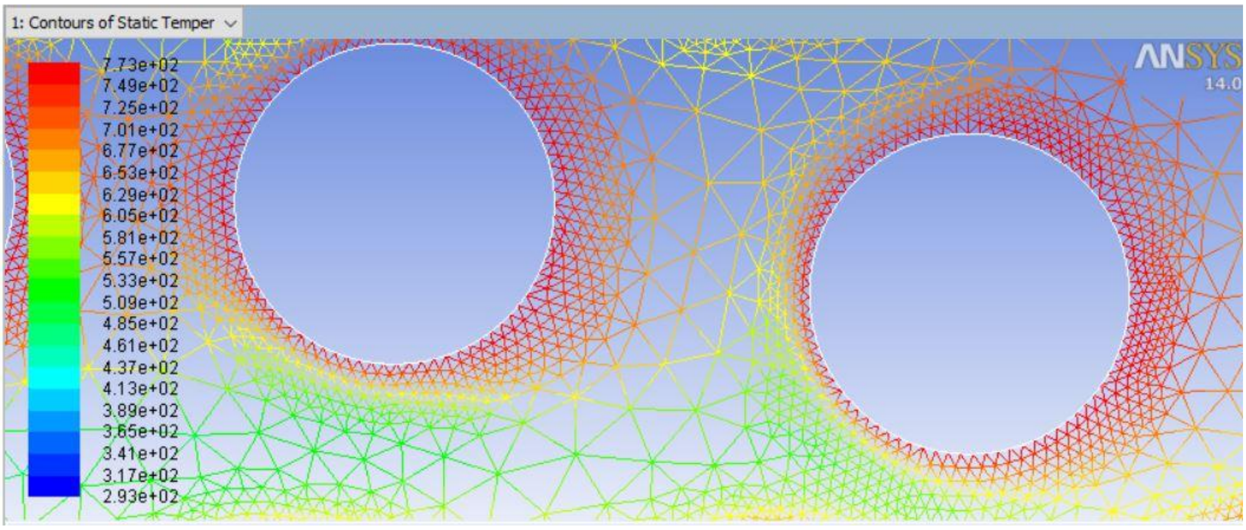

Figure 4.18: Temperature of the cylindrical surface (approx. 773K)

CHAPTER 5
CONCLUSION

We studied the Neutron flux distribution in a nuclear reactor with two important theories: Diffusion Theory and Boltzman theory. With Nuclear Distribution we were able study the neutron transport and collisions. Selecting the type of Reactor according to neutron flux distribution and source of neutron generation. Continuing it, we were able to understand the concept of multiplication factor which helped us in the criticality analysis.

We designed an algorithm that could calculate the criticality of the reactor according to the input parameters chosen from the user. It gave you the results whether the reactor is critical (k=1), subcritical (k<1) or supercritical (k>1).

After the neutron flux generation study we were able to understand the heat generation in the reactor. We were able to correlate how the neutron flux generated caused the temperature rise. Then we studied the methods of heat transfer within the fluid and from the solid fuel structure to the working fluid. We studied the theoretical calculations for the surface temperatures of the nuclear fuel rod, the surface temperature after the cladding and ambient temperature of fluid after heat transfer with both conduction and convection.

After the theoretical study of safe temperature limits we designed a 2-D model on ANSYS-CFD to understand the fluid flow temperature, pressure and velocity variations.

In the process of the whole project we learn the interdependence of neutron flux generated to the temperature and how the temperature would vary if there is a change in the criticality of the reactor. With the results we will be able to observe the safe operating parameters for a TRIGA MARK II power plant.